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Digital Control System Compensation to Obtain Continuous-System Operation

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by

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Digital
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ABSTRACT

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A compensation technique is described for improving the time response of the controlled variable for an incremental digital feedback control system. Compensation signals, supplied with the pulse input signal and generated within the system, provide a continuous reference-input signal, a continuous feedback signal, and a resulting continuous system error signal. The compensated system then exhibits the behavior of a continuous analog feedback control system rather than that of a discontinuous digital control system. However the compensated system retains the principal advantages of the uncompensated digital system such as an easily transmitted digital input signal and a very accurate digital component in the feedback signal; thus it maintains the digital accuracy of the uncompensated system while acquiring the "smoothness" of operation of a similar analog system.

An experimental model of a compensated digital control system is described. Experimentally obtained curves are also shown to compare the time response of the controlled variable with and without the compensation networks for several different input signals.

ACKNOWLEDGEMENTS

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LIST OF LITERAL SYMBOLS

A	Amplitude of compensated reference-input pulses R_{pc}; amplitude of constant voltage input to integrators I_1 and I_2
B	Amplitude of compensated reference-input pulses R_{pc} indicating a command to reverse direction or stop; $B > A$
E	System error
E_c	Continuous system error
$E(s)$	Laplace transform of system error
$e(\infty)$	Steady-state system error
FF	Flip-flop (bistable multivibrator)
$G(s)$	Transfer function of system actuator
I	Integrator employing an operational amplifier
K	Transfer-function gain
K_I	Integrator gain
R	Reference input
R_A	Approximated desired controlled-variable time response (R_D approximated by straight-line segments
R_c	Continuous reference input
R_d	Digital component of continuous reference input R_c

R_D Desired controlled-variable time response

R_Δ Incremental component of continuous reference input R_c

R_p Reference-input pulses to error bidirectional counter

R_{pc} Compensated reference-input pulses

R_{pk} Compensation pulses that compensate quantized reference-input pulses R_{pq} to produce R_{pc}

R_{pq} Quantized reference-input pulses; $R_{pq}+$ (and $R_{pq}-$) indicate a desired increase (and decrease) in θ

R(t) Continuous reference-input R_c at time t

R_w, R_x, R_y, R_z Digital component R_d at quanta point w, x, y, z, respectively

S Summing amplifier employing an operational amplifier

s Laplace operator

ST Schmitt trigger

T Actuator time-constant

t Time

ζ Damping ratio of actuator transfer function

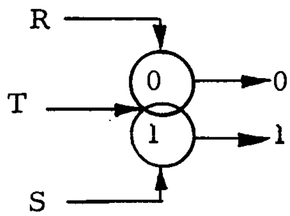
θ Controlled variable

$\dot{\theta}$ Controlled-variable velocity

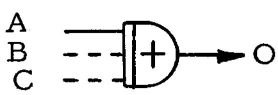
$\ddot{\theta}$	Controlled-variable acceleration
$\dddot{\theta}$	Time-rate of change of controlled-variable accel.
θ_c	Continuous controlled-variable feedback
θ_d	Digital component of continuous controlled-variable feedback θ_c
θ_Δ	Incremental analog component of continuous controlled-variable feedback θ_c
θ_p	Controlled-variable feedback pulses to error bidirectional counter
θ_{pq}	Controlled-variable quantized pulses from output of θ-quantizer
$\Delta\theta$	Increment of the controlled variable θ
$\theta(s)$	Laplace transform of controlled variable
$\dot{\theta}(s)$	Laplace transform of controlled-variable velocity
τ_k	Time interval between occurrence of compensating pulse R_{pk} and quantized pulse R_{pq} in the compensated reference input R_{pc}
τ_q	Time interval between the occurrence of two successive quantized R_{pq} pulses or between a R_{pq} pulse and a following stop-command pulse in R_{pc}
ω	Frequency

LIST OF LOGICAL SYMBOLS

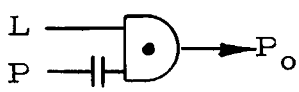
Symbol | Description



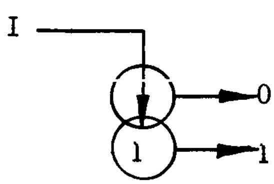
Bistable multivibrator
("Flip-flop")



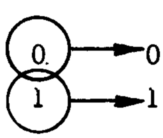
Nor gate



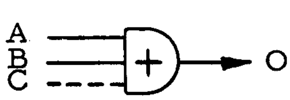
Gated pulse generator



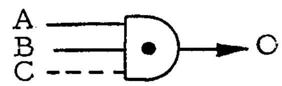
Monostable multivibrator
("one-shot")



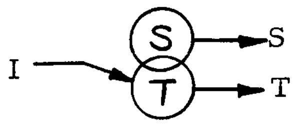
Astable multivibrator
("free-running")



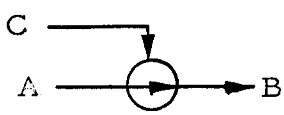
Or gate



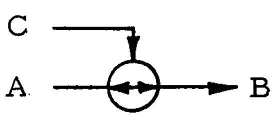
And gate



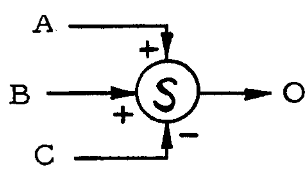
Schmitt trigger



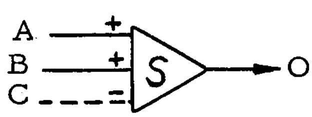
Gate (transistor)



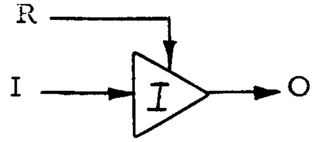
Relay (electromechanical)



Summing amplifier



Summing amplifier
(and inverter)



Integrator (and inverter)

$I \longrightarrow \boxed{ST} \longrightarrow O$ Schmitt trigger

CHAPTER I

INTRODUCTION

Incremental digital control systems are employed in many control applications because digital information is easily and accurately stored and transmitted. Digital pulses are used to supply input and feedback information to these systems. Incremental digital control systems are utilized to manipulate a controlled variable and its time derivatives along a prescribed path. However, at low input pulse rates, the controlled variable changes in a step-like manner instead of continuously; thus there are large errors in the controlled-variable displacement and rate.

The research work described here led to the development of compensation techniques for improving the time response of the controlled variable of an incremental digital control system. Compensation techniques are described which generate a continuous reference-input signal from pulse input information and which generate a continuous controlled-variable feedback signal from the quantized feedback pulses and the controlled-variable velocity signal. The summation of the continuous input signal and the continuous negative feedback signal provides a continuous error signal as the input to the system actuator, rather than the discontinuous error signal common to a typical uncompensated digital control system. Thus the compensated system retains the digital accuracy and other advantages characteristic of a digital system, and it also acquires the "smoothness" of operation as well as the analog resolution of a similar continuous analog control system.

CHAPTER II

INCREMENTAL DIGITAL CONTROL SYSTEM OPERATION

Description of the System

The block diagram of a typical incremental digital feedback position-control system is shown in Figure 2-1. Reference information R_A, consisting of the desired time response of the controlled variable θ, is quantized by the reference quantizer and stored (usually on magnetic tape or punched paper tape). The quantizing process consists of the generation of reference pulses; see Figure 2-2. Each pulse indicates that the magnitude of the controlled variable should change by one discrete increment (or quantum) during the period of time between the occurrence of that pulse and the occurrence of the succeeding pulse. If it is desired that the controlled variable should increase by one quantum, a plus pulse $R_{pq}+$ is generated; if it should decrease, a minus pulse $R_{pq}-$ is generated. Thus the frequency of the pulses is indicative of the desired rate of change of the controlled variable, and, at any given time, the

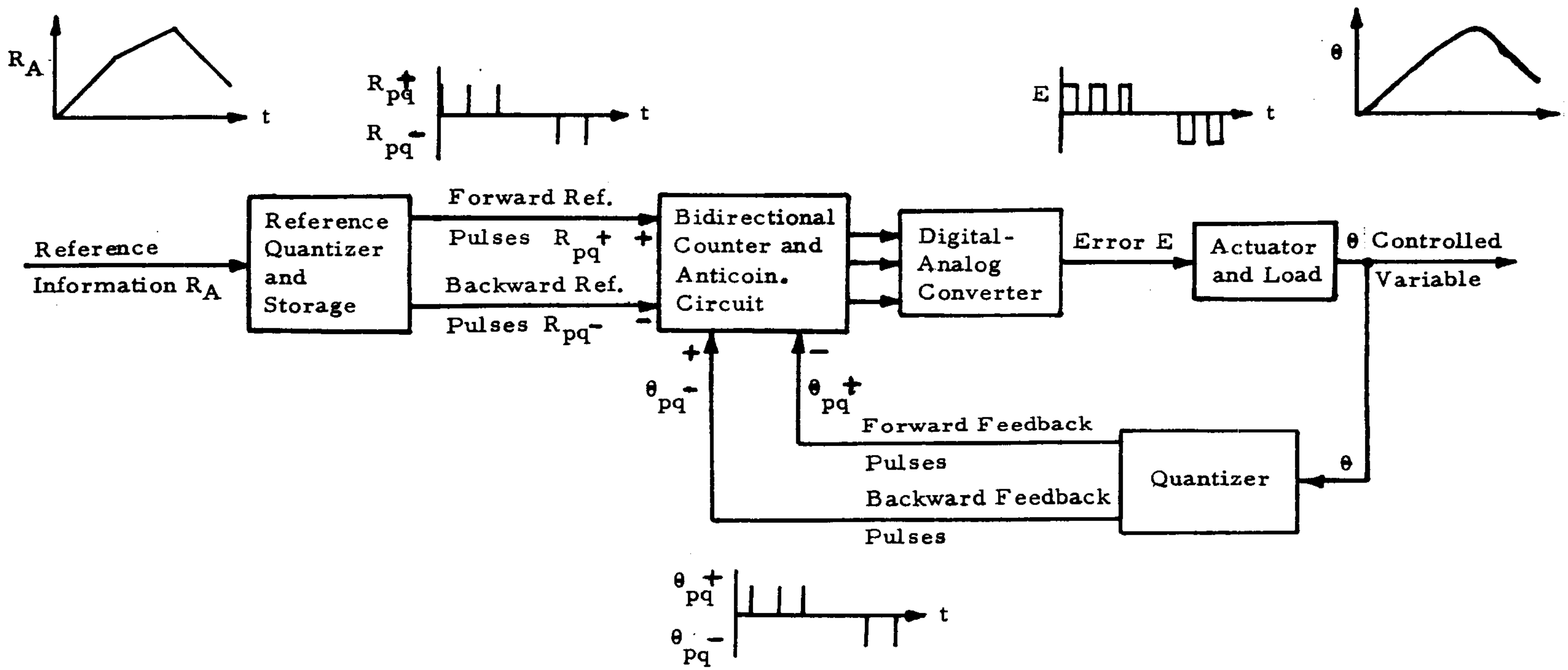


Figure 2-1. Block diagram of a typical incremental digital control system

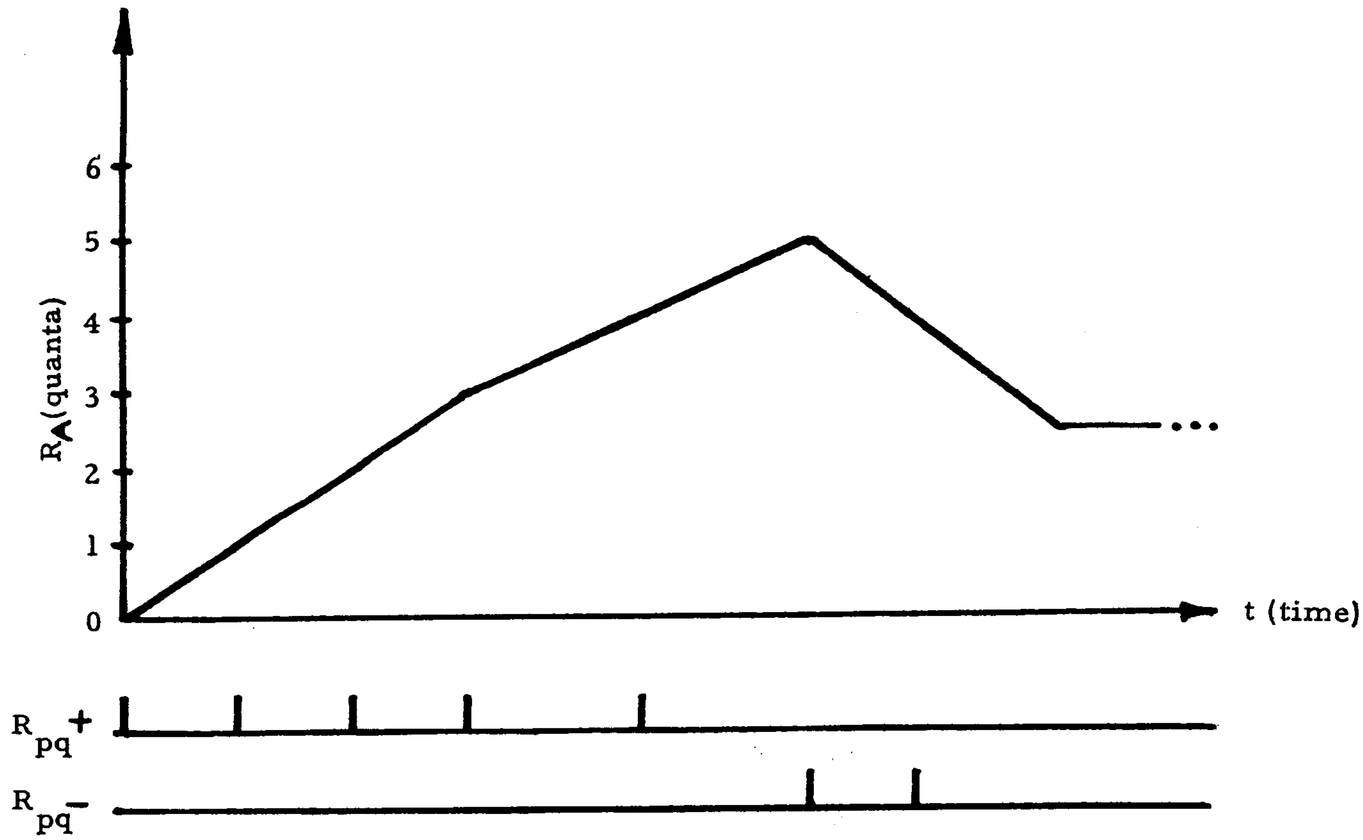


Figure 2-2. Quantization of reference-input information R_A

net algebraic sum of the generated pulses is indicative of the desired net change (measured in quanta) of the magnitude of the controlled variable.

These reference pulses are then supplied, in proper time, sequence and spacing, to a bidirectional counter; the receipt of a plus pulse increases, and a minus pulse decreases, the number in the counter. The number in the counter is representative of the instantaneous digital error existing between the desired value of the controlled variable and its actual value.

The digital-analog converter produces a discontinuously varying analog error signal E proportional to the digital error number in the bidirectional counter. This error signal is applied as the input to the system actuator and its mechanically coupled load.

The actuator alters its position θ in response to the error signal. For a typical control actuator, the direction and the average rate of change of θ are proportional to the polarity and the average magnitude, respectively, of the error signal.

The actuator is mechanically coupled to a quantizer. As the actuator alters its position θ, the quantizer emits a pulse each time that the magnitude of θ becomes equal to a quanta value. (A quanta value is an integral multiple of discrete equal increments of θ, called quanta, measured from some arbitrary reference position.) If θ is increasing, the emitted pulse will be a plus pulse $\theta_{pq}+$; if θ is decreasing, the emitted pulse will be a minus pulse $\theta_{pq}-$. Thus the total net number of pulses emitted is indicative of the net displacement of θ from the reference.

In order to provide negative feedback, the $\theta_{pq}+$ pulses are directed into the minus input and the $\theta_{pq}-$ pulses into the plus input of the bidirectional counter.

An anti-coincidence circuit is provided at the input to the bidirectional counter. This circuit prevents the simultaneous, or almost simultaneous, entrance of two pulses into the bidirectional counter and thus prevents the indeterminate counter operation that could result. The circuit delays the entrance of one of two simultaneously received pulses until the counter has responded to the other.

System Operation

Since all curves can be approximated by a series of straight-line segments, a typical desired time response for the controlled variable θ is a "ramp" response. A ramp response requires that θ change its magnitude by a specified number of quanta at a constant velocity $\dot{\theta}$. The required reference-input command for a ramp response consists of a series of pulses equally spaced in time. The total number of pulses represents the total desired quanta change in θ, and the pulse frequency represents the desired velocity in quanta per unit of time.

The ramp response of a typical digital position-control system is shown in Figure 2-3. Two undesirable characteristics can be observed:

(1) The actual steady-state response of θ deviates from the desired ramp response in a limit-cycle manner which produces a relatively large non-uniform position error.

(2) The steady-state velocity $\dot{\theta}$ is not constant as desired but also assumes a limit-cycle mode with large variations about the desired velocity.

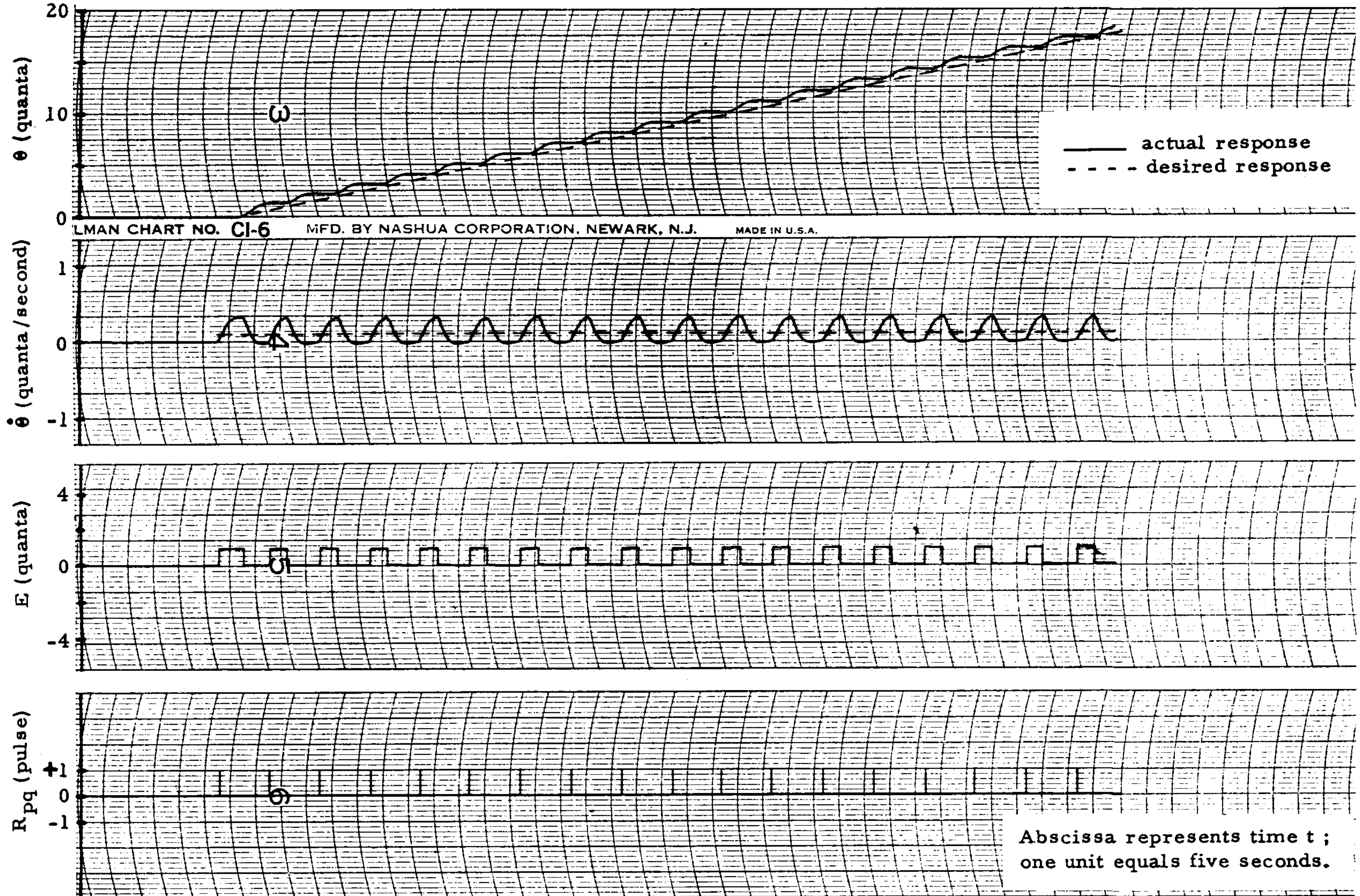


Figure 2-3. Ramp response of the controlled variable θ of a typical digital control system

These undesirable characteristics will be shown in Chapter III to be the result of the discontinuous error signals, illustrated in Figure 2-3, which are common to digital control systems.

For precision control a very small, uniform position-error and a uniform velocity are desirable; these operating characteristics are of particular importance for the control of machine tools in order to obtain an accurate, high-quality surface finish.

As can be seen in Figure 9-7b, the non-uniform time response of θ and $\dot{\theta}$ to ramp commands is most pronounced when the input pulse period is long when compared with the step-response time of the actuator. Then steady-state system operation can be qualitatively described as follows. When a reference-input pulse is received, a one-quanta error signal is applied to the actuator. The actuator accelerates and changes its position θ by one quanta and the resulting feed-back pulse decreases the error signal to zero. With zero input, the actuator decelerates toward a rest condition. When the next input pulse is received, the above operation is repeated. The time response of θ then assumes a step-like mode.

The limit-cycle deviation of θ from a true ramp response is also a function of quanta size; the smaller the increment of θ designated as a quanta, the smaller the deviation. However, a practical limit is placed on the minimum possible quanta size because of the precision and cost of the feedback quantizer required and due to the undesirable effects produced by mechanical vibrations.

CHAPTER III

CONTINUOUS CONSTANT ERROR SIGNAL REQUIRED FOR CONSTANT CONTROLLED-VARIABLE VELOCITY

It will now be shown that, for an actuator which can be easily modeled by a linear differential equation, a constant input error-signal is required in order to obtain a constant steady-state controlled-variable velocity.

The dominant transfer function G(s) for the small signal operation of a typical system actuator, such as an ac servomotor or a hydraulic servomotor is of the form

$$G(s) = \frac{K}{s(T_1 s + 1)(T_2 s + 1)}$$

which contains a single integration. The Laplace transform of the controlled variable $\theta(s)$ is

$$\theta(s) = E(s)\, G(s)$$

where E(s) is the Laplace transform of the error-signal input to the actuator. Then the Laplace transform of the controlled-variable velocity $\dot{\theta}(s)$ is

$$\dot{\theta}(s) = s\theta(s) = sE(s)\,G(s) = \frac{sE(s)\,K}{s(T_1 s + 1)(T_2 s + 1)} \quad .$$

Solving for E(s) gives

$$E(s) = \frac{\dot{\theta}(s)}{K}\,(T_1 s + 1)(T_2 s + 1) \ .$$

Now for a constant output velocity $\dot{\theta}(s) = \frac{\dot{\theta}}{s}$ and

$$E(s) = \frac{\dot{\theta}}{Ks}\,(T_1 s + 1)(T_2 s + 1) \quad .$$

Then the application of the final-value theorem gives the steady-state relationship between the desired constant output velocity $\dot{\theta}$ and the required system error signal $e(\infty)$.

$$e(\infty) = \Big[sE(s)\Big]_{s=0} = \frac{\theta}{K}$$

Thus it is seen that a constant output velocity requires a constant error signal.

Now an uncompensated digital system's digital-analog converter is capable of supplying error signals of integral magnitude only; see Figure 2-3. When a velocity is desired that requires a non-integral error signal, the system will assume a mode of operation such that the output of the digital-analog converter will alternate between two (or more) integral values in a periodic manner so as to produce an average output equal to the required error signal. This periodic discontinuous error-signal input to the actuator produces a periodic output velocity having an average value equal to the desired value.

It is thus apparent that, in order to obtain a constant uniform steady-state output velocity from a digital control system, it is necessary to compensate the system so as to obtain a continuous constant error-signal input to the system actuator. In order to obtain this continuous error signal E_c, it is necessary to first generate a continuous reference-input signal R_c and a continuous controlled-variable feedback signal θ_c since $E_c = R_c - \theta_c$.

CHAPTER IV

GENERATION OF A CONTINUOUS REFERENCE INPUT SIGNAL

Approximation of the Desired Output

A segment of a simple desired controlled-variable time-response curve R_D is shown in Figure 4-1. This curve R_D is approximated by a curve R_A which consists of straight-line segments extending between the quanta-point crossings of R_D. Hereafter R_A will be considered as the desired controlled-variable time-response curve, and thus it will also be considered as the reference-input signal that must be supplied to the system to command the controlled variable.

Quantization of the Approximated Desired-Output

In an uncompensated digital control system, R_A would be quantized into reference-input pulses $R_{pq}+$ and $R_{pq}-$; the occurrence of a $R_{pq}+$ (or a $R_{pq}-$) pulse would indicate that the controlled variable θ should increase (or decrease) in

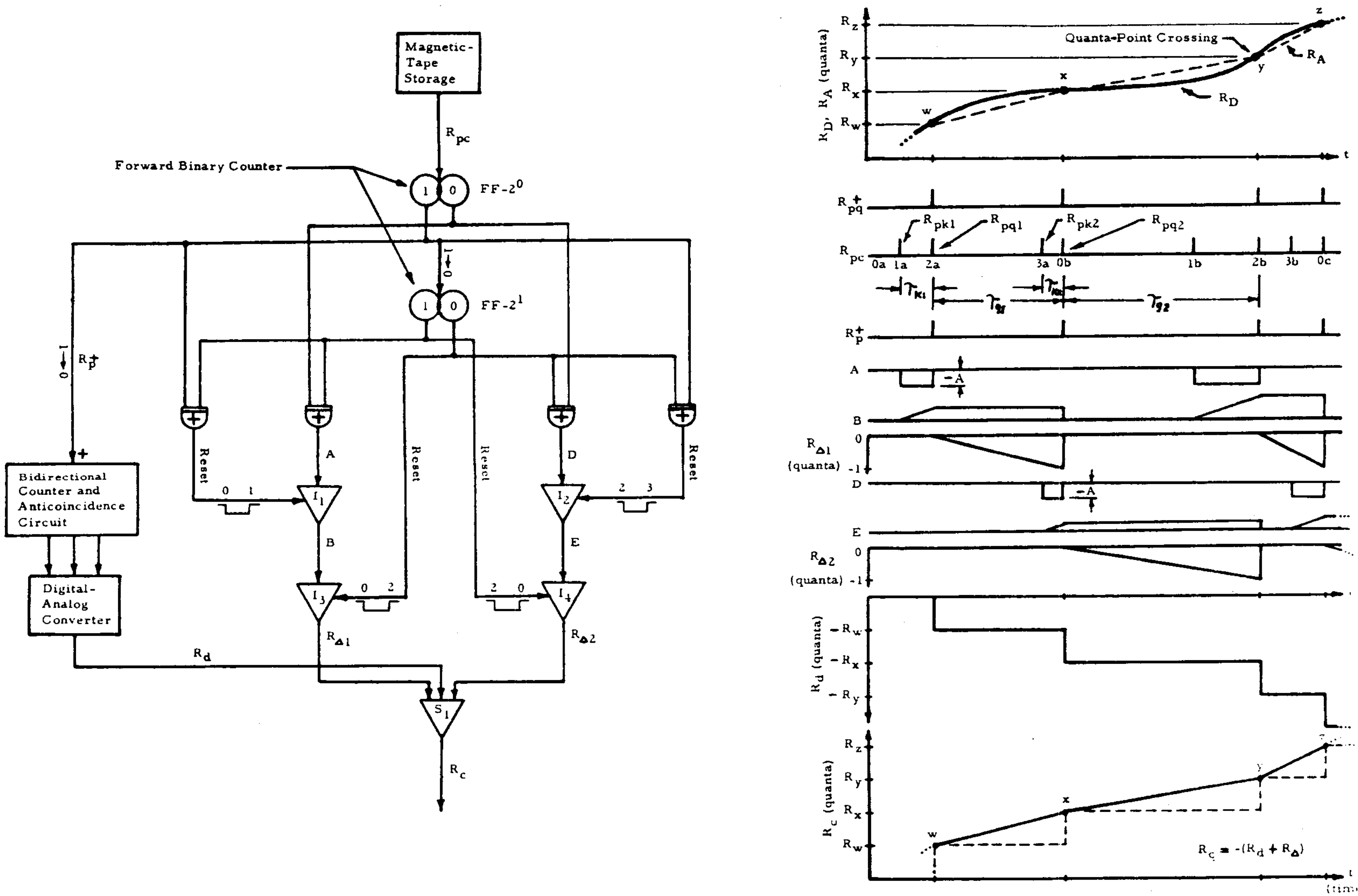


Figure 4-1. Signal-flow diagram in a reference-input compensation circuit

magnitude by one quanta during the period of time between the occurrence of that pulse and the occurrence of the succeeding pulse.

Compensation Pulses

The reference-input compensation is generated from information obtained by the addition of a compensation pulse R_{pk} which precedes each R_{pq} pulse by a variable time interval τ_k. The interval τ_k is made directly proportional to the controlled-variable velocity θ (the slope of the R_A-t approximation curve) that is desired during the time-interval τ_q between the occurrence of that R_{pq} pulse and the occurrence of the succeeding R_{pq} pulse.

The calculation of the proper time-location of these compensation pulses is described below for the typical desired controlled-variable response R_A shown in Figure 4-2. Assume that it is desired to generate the continuous linear input voltage-ramp (measured in quanta) that must occur during the time interval between t_a and t_b, that is, during the interval between the desired crossing of quanta-point "a" and the crossing of the successive quanta-point "b". From the point-slope form of

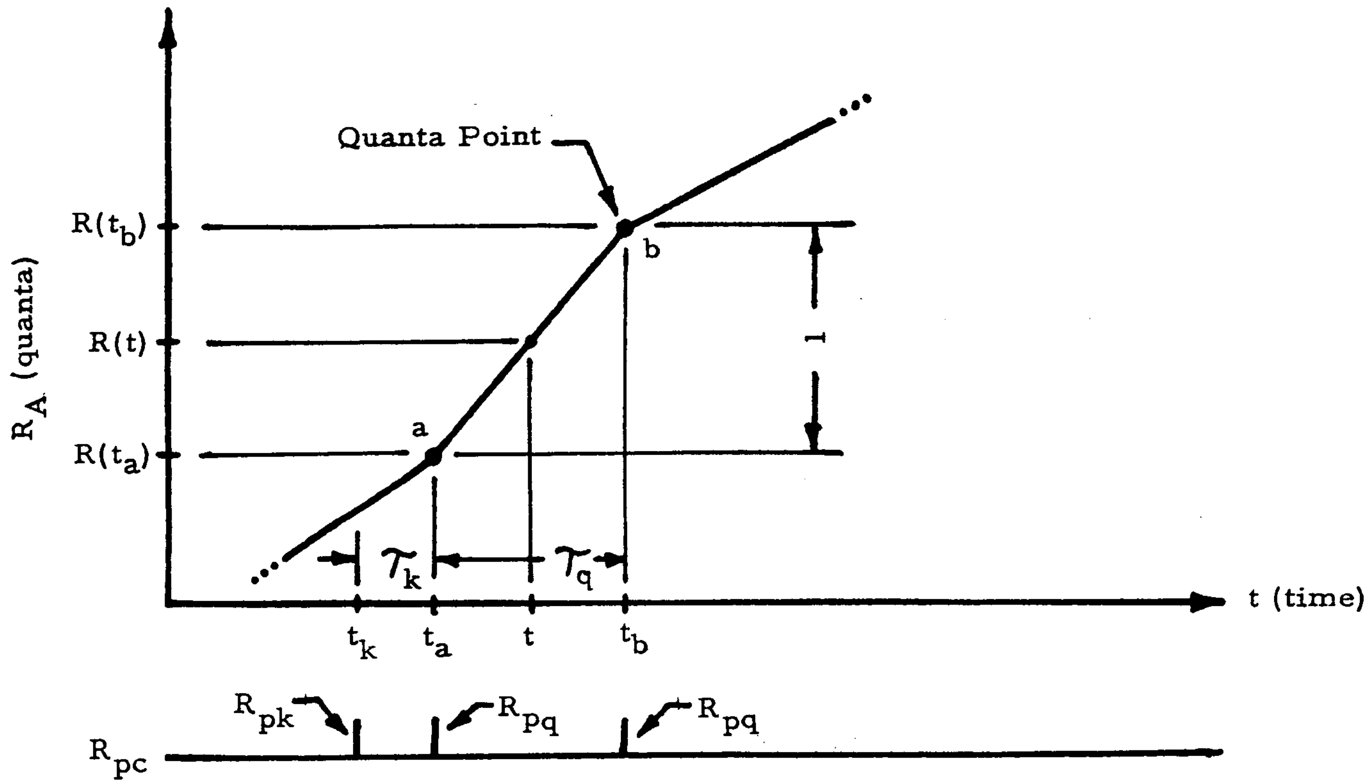


Figure 4-2. A segment of a typical controlled-variable time-response curve

the equation of a line, the voltage $R(t)$ at time t can be found for $t_a \leq t \leq t_b$.

$$\frac{R(t) - R(t_a)}{t - t_a} = \frac{R(t_b) - R(t_a)}{t_b - t_a} = \frac{1}{\tau_q} = \dot{\theta}_{\tau_q}$$

$$R(t) = R(t_a) + \frac{1}{\tau_q}(t - t_a)$$

It can be seen that the third term in this expression could be obtained by performing the integration of a constant voltage proportional to $1/\tau_q$ during the interval from t_a to t. Then

$$R(t) = R(t_a) + \int_{t_a}^{t} \frac{1}{\tau_q}\, dt \quad .$$

Now it is desired to have available, at time t_a, a constant voltage proportional to $1/\tau_q$. It can be seen also that, if another constant voltage "A" is made available for integration during the interval from a preceding time t_k (where $t_k = t_a - \tau_k$) to t_a, the following result will occur.

$$\int_{t_k}^{t_a} A\,dt = A(t_a - t_k) = A\tau_k$$

If $A\tau_k$ is to be equal to $1/\tau_q$, then

$$A\tau_k = \frac{1}{\tau_q}$$

and

$$\tau_k = \frac{1}{A\tau_q} = \frac{\theta}{A}$$

Thus if a compensation pulse R_{pk}, which precedes the quantized pulse R_{pq} at t_a by the interval τ_k, initiates the integration of the constant voltage A and if the R_{pq} pulse terminates this integration and initiates the integration of the result $A\tau_k$, which continues during the interval τ_q, then the desired voltage ramp R_Δ can be generated between time t_a and t_b. That is,

$$R(t) = R(t_a) + \int_{t_a}^{t} \left[\int_{t_k}^{t_a} A\,dt \right] dt$$

which is

$$R(t) = R_d + R_\Delta = R_c$$

for $t_a \leq t \leq t_b$.

The digital component R_d of the continuous reference input R_c is the component of the output of the system error digital-analog converter due to the input of the quantized reference-input pulses R_p to the bidirectional counter. The incremental component R_Δ is the compensating ramp component; its generation is described below.

Generation of the Continuous Reference-Input Signal

The generation of the continuous reference-input signal R_c, from the compensated reference-input pulses R_{pc} can be explained by further reference to Figure 4-1.

Consider the generation of the straight-line segments wx, xy, and yz which compose the illustrated portion of the curve R_A. Assume that the flip-flops (FF-2^0 and FF-2^1) of the forward binary counter are in the 0-state at time 0a preceding the reception of the first illustrated compensation pulse R_{pk1} at time 1a.

During the interval τ_{k1} following the R_{pk1} pulse, integrator I_1 integrates its input -A and holds the result $A\tau_{k1}$ during the following interval τ_{q1}. (Note that the operational amplifiers comprising the integrators and the summer produce an algebraic sign inversion.) At time 2a, the R_{pq1} quantized pulse directs a R_p+ pulse into the bidirectional counter, and the associated digital-analog converter produces an output R_d of $-R_w$. During the interval τ_{q1}, the output $A\tau_{k1}$ of integrator I_1 is integrated by integrator I_3 giving the desired R_Δ output of $-(t-2a)(A\tau_{k1})$. Now since τ_{k1} was made equal to $1/A\tau_{q1}$ to provide proper time orientation of the R_{pk1} compensation pulse, then

$$R_\Delta = \frac{-(t-2a)}{\tau_{q1}}$$

during the interval τ_{q1} where $2a \leq t \leq 0b$. Thus during the interval τ_{q1}, the continuous reference input R_c is obtained by the summation (by summing amplifier S_1) of this linearly-varying component R_Δ and the reference-input component R_d of the output of the system error bidirectional counter and digital-analog converter. That is,

$$R_c = -(R_d + R_\Delta) = R_w + \frac{(t-2a)}{\tau_{q1}}$$

for $2a \leq t \leq 0b$. At time 0b, $t-2a = \tau_{q1}$ and so $R_c = R_w + 1 = R_x$; then integrators I_1 and I_3 are reset to zero initial-conditions and an R_p+ pulse is directed into the plus input of the bidirectional counter by the reception of the R_{pq2} quantized pulse. This R_p+ pulse increases the digital-analog converter output by one quanta to R_x. The next R_Δ ramp is generated in a similar manner by integrators I_2 and I_4.

Note that at time 3a, an operation similar to that described above was initiated for integrators I_2 and I_4; thus the two sets of integrators alternately supply the R_Δ component of R_c.

It has been shown above that the reference-input compensation consists of the generation of an incremental ramp-component R_Δ. R_Δ increases linearly from a magnitude of zero to a magnitude of one quanta during the interval between the reception of two successive quantized pulses R_{pq}. When a R_{pq} pulse is received, the R_Δ component is reset to zero and an R_p pulse enters the bidirectional counter, changing the d-a output by one quanta and the process is repeated. Thus a continuous reference input signal R_c is generated by the addition of the linearly-varying incremental ramp component R_Δ and the step-varying digital component R_d.

Generation of the R_{pk} Pulses

In the experimental model of a compensated digital system described in Chapter VIII, the compensation pulses R_{pk} and the quantized pulses R_{pq} were recorded on magnetic tape to provide the compensated reference-input pulses R_{pc}. Since relatively slow input-pulse rates were desired, a push-button and stop-watch technique was used in the recording of the pulses. However for high input-pulse rates, a recording process should be employed that would be more rapid and more accurate.

One technique that could be used consists of a punched paper tape input and an electronic decoding circuit. The tape would be coded with information describing the desired change in magnitude and the desired rate of change of the controlled variable for each individual ramp (straight-line segment) approximating the desired controlled-variable time response. This information would then be decoded by the decoding circuit which would then generate the required compensated reference-input pulses R_{pc}.

CHAPTER V

GENERATION OF A CONTINUOUS CONTROLLED-VARIABLE FEEDBACK SIGNAL

A typical electromechanical digital position-control system will be assumed for the following discussion. The actuator, the controlled variable, and the feedback paths for the compensated digital control system are shown in Figure 5-1. Figure 5-2 shows a typical time response for the controlled variable θ (position) and for the θ-feedback-signal components.

The actuator, consisting of an electric motor supplied by a voltage amplifier, etc., is mechanically coupled to a quantizer and a dc tachometer. Operation of the actuator causes a change in the magnitude of the controlled variable θ.

The quantizer operates in the following manner. (1) The total possible operating range of θ has been subdivided into preselected equal-increments called "quanta"; the points of separation between these increments are designated "quanta points". (2) At the instant that θ, having a positive velocity,

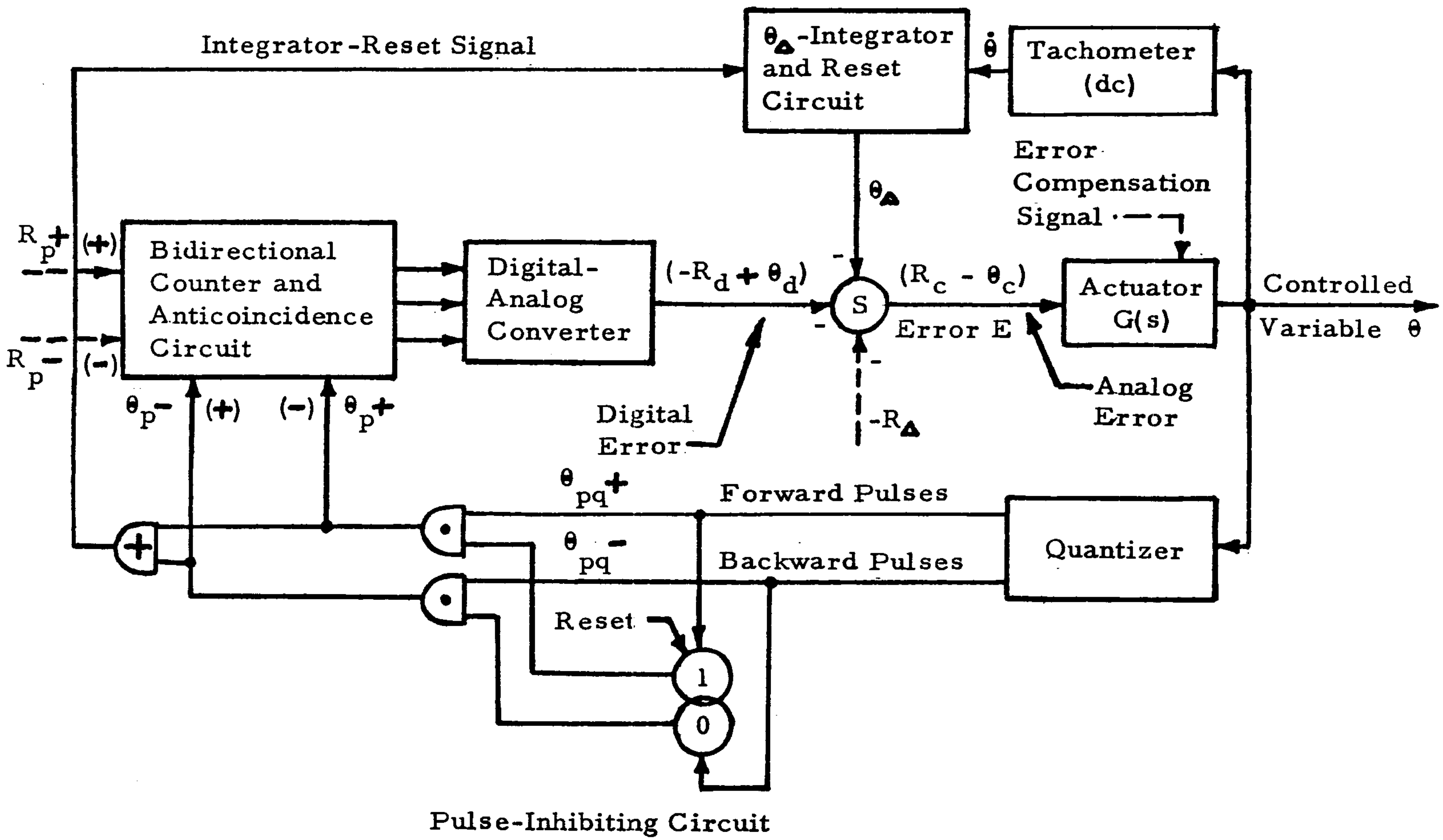


Figure 5-1. Actuator, controlled-variable, and feedback paths for the compensated digital control system

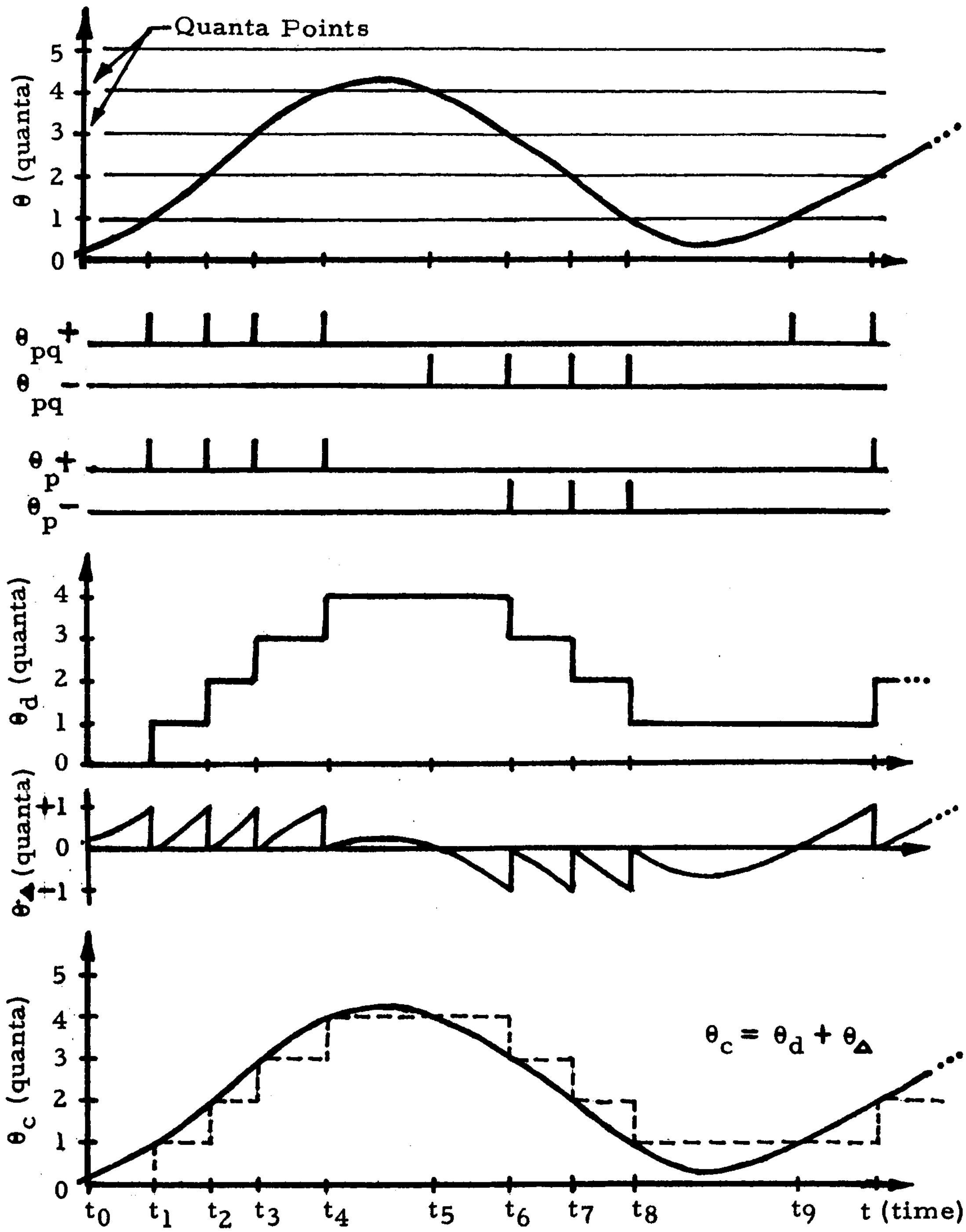


Figure 5-2. Typical time-response for the controlled variable θ and the θ-feedback components

increases through a quanta-point value, a forward pulse $\theta_{pq}+$ is generated; likewise, when θ, having a negative velocity, decreases through a quanta-point value, a backward pulse $\theta_{pq}-$ is generated. Thus the algebraic net number of θ_{pq} pulses generated during a specified time interval is indicative of the net change in quanta of the magnitude of θ during that interval.

The dc tachometer generates a dc voltage which has a magnitude proportional to the time-rate of change $\dot{\theta}$ (velocity) of the controlled variable and a polarity identical to the algebraic sign of $\dot{\theta}$.

The input to the actuator consists of a position-error signal E and any error-compensation signals that might also be provided. The error signal can be considered to consist of the algebraic summation of two components, the compensated reference-input signal R_c (described in Chapter IV) and the compensated negative feedback signal $-\theta_c$. Now θ_c also consists of the algebraic summation of two components, a digital step-varying component θ_d and an incremental analog component θ_Δ which is "smoothly-varying" within incremental segments; see Figure 5-2.

The digital component θ_d is generated by feeding the θ_p pulses obtained from the quantizer output into a bidirectional counter and digital-analog converter combination. In order to obtain negative feedback, the forward pulses θ_p+, indicating an increase in θ, are directed to the minus input of the counter and thus decrease the number in the counter; the backward pulses θ_p-, indicating a decrease in θ, are directed to the plus input of the counter and increase the number in the counter. The digital-analog converter then produces an output voltage proportional to the number in the counter; since this digital number changes in a step-like manner, the output of the converter also has a step-like time-variation. Due to the characteristics of the θ_Δ signal, a "pulse-inhibiting" circuit is required in the feedback path between the quantizer and the bidirectional counter; this circuit is explained later.

The incremental component θ_Δ is generated by feeding the dc tachometer output signal $\dot{\theta}$ into an integrator and reset circuit. The integrator integrates this velocity signal $\dot{\theta}$ and gives an output θ_Δ which has a magnitude and sign proportional to the algebraic net change in the controlled variable θ which has occurred since the previous quanta point was crossed. Since $\dot{\theta}$, and thus θ_Δ, can assume both positive and negative values, it is only necessary to reset the integrator and to change the number in the counter each time θ_Δ attains an absolute magnitude equivalent to one quanta. (Observe θ, θ_Δ, θ_d, and θ_p during the time interval between t_4 and t_6 in Figure 5-2.) The reset of the integrator and the change of the number in the counter by one quanta unit are performed each time that a θ_p pulse occurs; a θ_p pulse is a θ_{pq} pulse which has been allowed to pass through the pulse-inhibiting circuit.

The pulse-inhibiting circuit compares the algebraic sign of each θ_{pq} pulse with that of the previous θ_{pq} pulse; if the signs are alike it passes the pulse as a θ_p pulse; if they are unlike, it inhibits it. Thus θ_{pq} pulses, which are generated due to successive crossing of the same quanta point, are inhibited from appearing as θ_p pulses.

The compensated negative feedback signal $-\theta_c$ is the sum of the inverted digital component $-\theta_d$ and the inverted incremental component $-\theta_\Delta$; these components and their sum are shown in Figure 5-2. (Note that θ_d was inverted previously by feeding the θ_p+ pulses into the minus input of the counter and the θ_p- pulses into the plus input; then the digital-analog converter, which produces a negative output, inverted $-\theta_d$ again giving θ_d as the θ-component of its output.) Thus a continuous negative feedback signal $-\theta_c$ has been produced. For example, in the interval $t_4 \leq t \leq t_6$

$$-\theta_c = -\theta_d - \theta_\Delta$$

$$-\theta_c = -\theta(t_4) - \int_{t_4}^{t} \theta \, dt$$

It can be seen that, since the θ_Δ-integrator is reset and the number in the counter is changed coincident with the occurrence of each θ_p pulse (which occurs when θ crosses a different quanta point), the positional accuracy of the feedback signal is equivalent to that of an uncompensated digital system at the instant when θ crosses quanta points, and it is much better between quanta points. Also the resetting of the integrator with each θ_p pulse prevents an accumulation of any error in θ_Δ that might exist due to small inaccuracies in the integrator gain or in the tachometer output.

It should be noted here that only one bidirectional counter and digital-analog converter combination is required to generate both the reference-input digital component R_d and the controlled-variable digital feedback component θ_d. The combination of these components occurs as the negative output of the system error bidirectional counter and digital-analog converter combination in the form of $-(R_d - \theta_d)$.

CHAPTER VI

PROPORTIONAL-PLUS-INTEGRAL ERROR COMPENSATION

It will now be shown that the addition of proportional-plus-integral error compensation will reduce the steady-state system error of the compensated digital control system to zero for ramp-input commands (constant-velocity commands).

Consider a typical actuator transfer function G(s), as assumed in Chapter III, which was

$$G(s) = \frac{K}{s(T_1 s + 1)(T_2 s + 1)} \quad .$$

Now with proportional-plus-integral compensation, the Laplace transform of the input signal to the actuator is

$$E(s) + K_I \frac{E(s)}{s} = E(s)\left(1 + \frac{K_I}{s}\right) \quad ,$$

and the Laplace transforms of the controlled variable $\theta(s)$ and its velocity $\dot{\theta}(s)$ are

$$\theta(s) = E(s)\left(1 + \frac{K_I}{s}\right)\frac{K}{s(T_1 s + 1)(T_2 s + 1)}$$

$$\dot{\theta}(s) = s\theta(s) = \frac{KE(s)(s + K_I)}{s(T_1 s + 1)(T_2 s + 1)} \quad .$$

Solving for E(s) for a constant desired velocity of $\dot{\theta}(s) = \frac{\dot{\theta}}{s}$ gives

$$E(s) = \frac{\dot{\theta}(T_1 s + 1)(T_2 s + 1)}{K(s + K_I)} \quad .$$

Then the application of the final-value theorem gives the steady-state relationship between the desired constant output velocity $\dot{\theta}$ and the required system error signal $e(\infty)$.

$$e(\infty) = \Big[sE(s)\Big]_{s=0} = 0$$

Thus if proportional-plus-integral error compensation is employed, with the compensated digital system, the steady-state system error is decreased to zero for ramp-input commands.

It should be noted, however, that if proportional-plus-integral error compensation is used with an uncompensated digital system, a low-frequency, low-amplitude limit cycle may occur for the controlled variable. The occurrence of this limit-cycle operation can be understood by reference to a Nyquist diagram. It was shown by Taft[1*] that a describing-function approximation of the discontinuous elements in the feedback loop of a digital system produced a locus for the "critical-point" which lay along the negative real axis between -0.7 and $-\infty$. Now if the transfer function G(s) of the system actuator contains a single integration (a type-1 system), then the application of proportional-plus-integral error compensation will convert G(s) into an equivalent type-2 transfer function G'(s) which contains a double integration. For very low frequencies (ω) the locus of $G'(j\omega)$ would also approach the negative real axis of the Nyquist diagram. Since the describing function is only an <u>approximation</u> of the discontinuous

*The superscript numerals refer to Bibliography.

elements comprising the system feedback loop, intersection of the $G'(j\omega)$ locus and the critical-point locus may occur for low frequencies. Thus a low-frequency, low-amplitude limit cycle would occur for the controlled variable.

CHAPTER VII

ADVANTAGES OF THE COMPENSATED DIGITAL CONTROL SYSTEM

The compensated digital control system has the following characteristics which may be considered as advantages when compared with those of the uncompensated system.

(a) The compensated system retains the pulse-data mode of input operation of the uncompensated system in which all reference-input information is received in pulse form. (Pulse information can usually be more easily and accurately stored and transmitted than the corresponding analog information.) However, the compensation circuits in the input channel of the system convert this input pulse-data into an equivalent continuous reference-input signal; this signal has a form identical to that of the unquantized reference-input signal which represented the desired time response of the controlled variable. Thus the reference-input component of the error signal supplied to the system actuator is continuous and has the accuracy of the uncompensated digital input signal at the time of the reception of its input pulses and an accuracy that is much better during the time interval between these pulses.

(b) The compensated system retains the pulse-data mode of feedback operation of the uncompensated system in which the principal component of the controlled-variable feedback information is in digital pulse form from which a digital feedback component is derived. However, the compensation circuits in the feedback loop also generate an additional incremental analog feedback component during the time interval between the occurrence of feedback pulses. By the addition of these digital and incremental components, a continuous feedback signal is obtained which has a form identical to that of the response of the controlled variable. Thus the feedback component of the error signal supplied to the system actuator is continuous and has the accuracy of the uncompensated digital feedback signal at the time of the generation of feedback pulses and an accuracy that is much better during the time interval between these pulses.

(c) A continuous system error signal is produced by the summation of the continuous reference-input component and the continuous negative feedback component. This continuous error signal provides a "smooth" continuous input signal to the system actuator, rather than the discontinuous "step-like" error signal which occurs in an uncompensated digital control system. This continuous error signal makes the internal operation of the compensated digital control system similar to that of a continuous analog system.

(d) The digital system has been compensated so that it behaves internally like a continuous system; therefore, it has the following steady-state operating characteristics for a typical actuator transfer-function having a single integration (a type-1 system) with a ramp input signal (a constant controlled-variable-velocity command).

1. The controlled variable follows the ramp input command with a constant position-error and a constant velocity.
2. The position-error will be zero with proportional-plus-integral error compensation.
3. A larger system-gain can be used for third- or higher-order systems since the critical point on the Nyquist

diagram is located at -1.0 on the real axis rather than at -0.7 which was shown in a describing-function analysis by Taft[1*] to be the case for an uncompensated digital control system; the larger gain will reduce the position error.

(e) Analog signals, as well as digital signals, are available within the system for use in additional transient-compensation techniques. These signals may be used for improving the controlled-variable response during the transient period that follows a system velocity-change command. A possible compensation technique could employ "digital-lead" compensation (as described by Gaither[2]) in combination with analog compensation and utilize a switching circuit to alter the compensation as a function of system-error magnitude.

(f) When no input signal or load disturbance exists, the compensated system comes to rest with the controlled variable coincident with a quanta-point location due to the corrective action produced by the analog compensation component of the

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*The superscript numerals refer to the Bibliography.

negative feedback signal. Thus the steady-state rest-position of the controlled variable is always accurately known for the compensated system while it may be located at any unknown position between two quanta points for the uncompensated system.

(g) Since the system was compensated so that it behaves internally as a continuous system, the many well-known analysis and synthesis techniques employed with continuous systems may also be used in the design or study of the compensated digital system.

(h) Also the continuous-system behavior of the compensated system and its error signal prevent the occurrence of the limit-cycle behavior that is common to uncompensated digital control systems due to ramp inputs, load disturbances, and drift. This limit-cycle behavior is eliminated because the compensated system is capable of supplying to the actuator an analog error signal that can assume the correct magnitude required for system equilibrium. The error signal for an uncompensated system is limited to integral values;

thus when a non-integral magnitude is required, it must vary discontinuously between two (or more) integral values in order to supply a signal having an equivalent average value.

CHAPTER VIII

DESCRIPTION OF AN EXPERIMENTAL MODEL OF A COMPENSATED DIGITAL CONTROL SYSTEM

A model of a typical digital position-control system, compensated to provide a continuous reference-input signal, a continuous controlled-variable feedback signal, and thus a resulting continuous system error signal, was constructed; see Figures 8-1 and 8-2. The response of this system, with and without the presence of the compensation signals, demonstrates the improvement in system operation that can be obtained when the compensation technique is employed.

Reference-Input Circuit

The reference-input circuit, which generates the continuous reference-input signal R_c from information obtained from its compensated pulse-input signal R_{pc}, will be described by considering its response to a typical input.

A simple desired controlled-variable time-response curve R_D is shown in Figure 8-3. As stated in Chapter IV, the curve

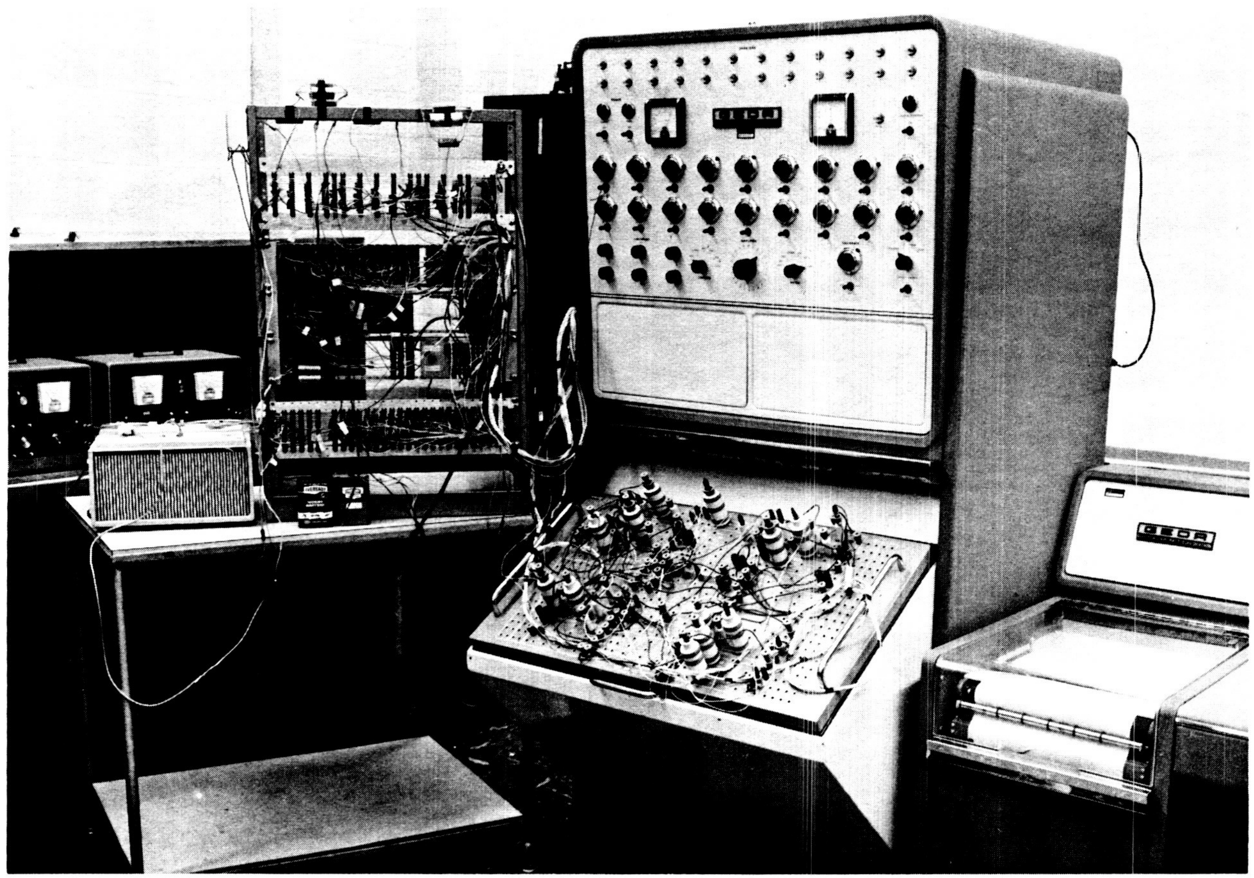


Figure 8-1. Photograph of the experimental model of a compensated digital control system

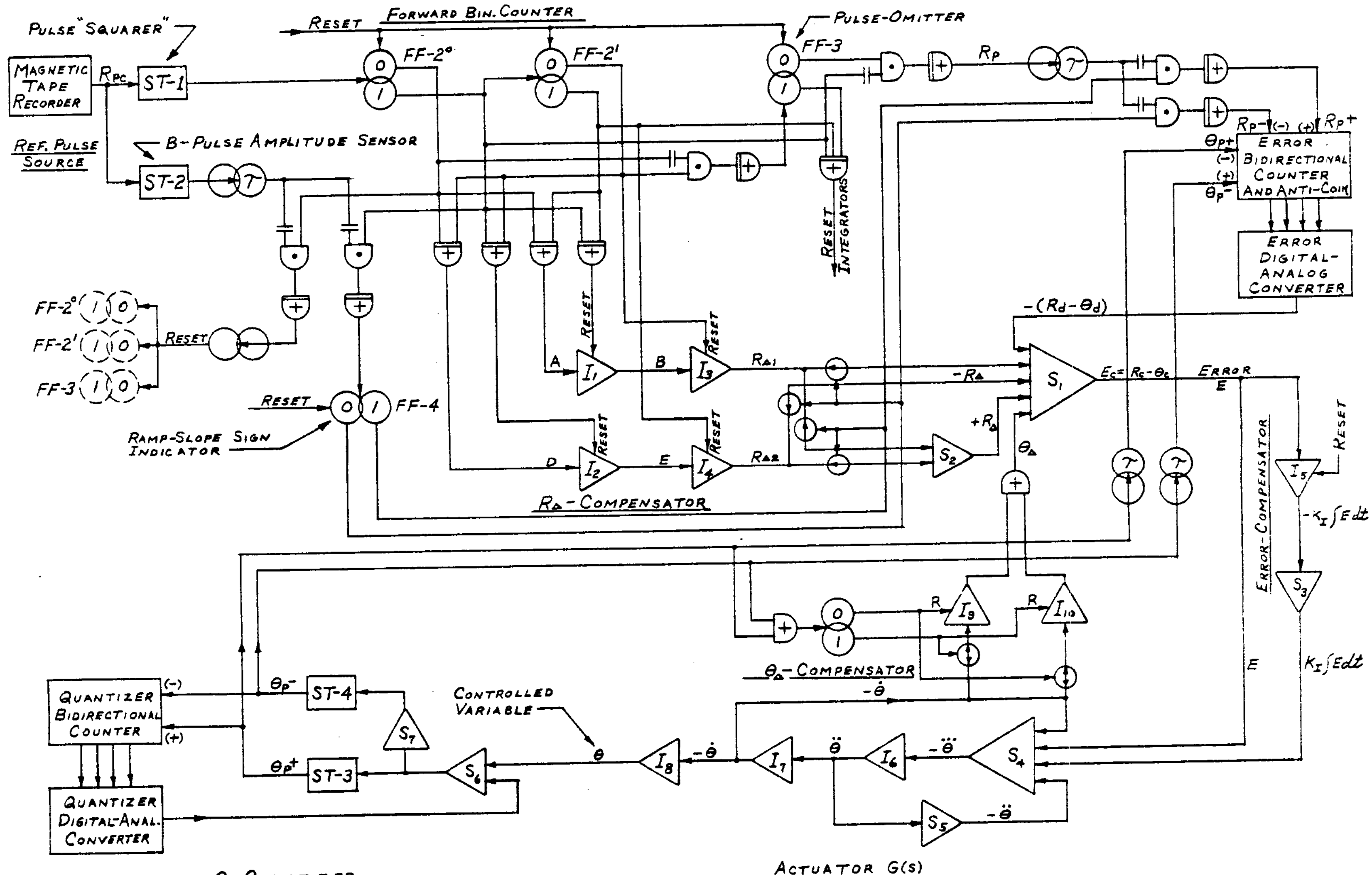


Figure 8-2. Circuit diagram of the experimental model of the compensated system

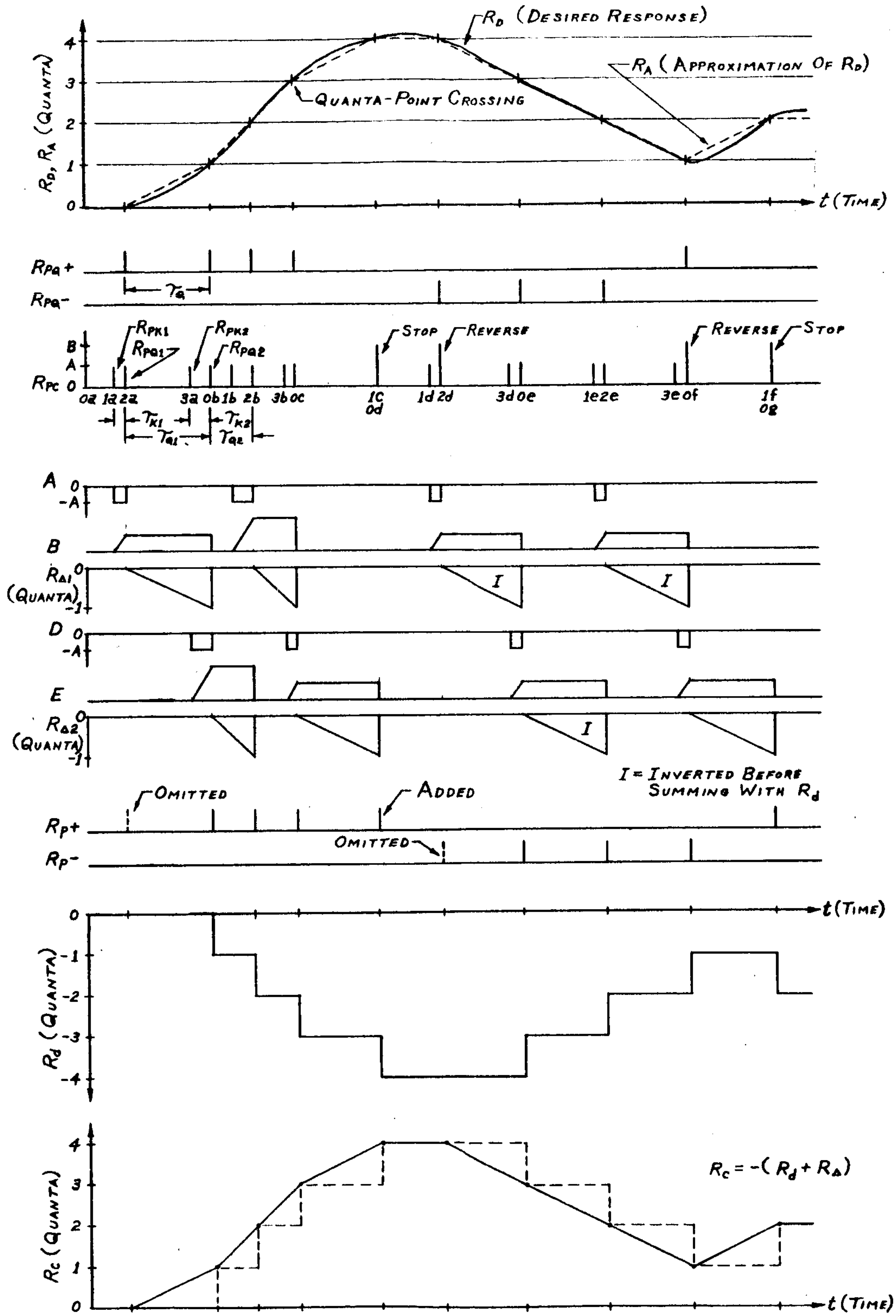


Figure 8-3. Signal-flow diagram in the experimental model of the compensated system

R_D is approximated by a curve R_A consisting of straight-line segments extending between the quanta-point crossings of R_D. Hereafter, R_A will be considered as the desired controlled-variable time-response curve and thus as the reference-input signal required to command the controlled variable.

In an uncompensated digital control system, R_A would be quantized into reference input pulses $R_{pq}+$ and $R_{pq}-$. The occurrence of a $R_{pq}+$ (or a $R_{pq}-$) pulse would indicate that the controlled variable θ should increase (or decrease) in magnitude by one quanta during the period of time τ_q between the occurrence of that pulse and the occurrence of the succeeding pulse.

In the compensated control system to be described, only one input-signal channel is required. The compensated pulse-input signal R_{pc} to this channel consists of pulses of amplitude "A" and pulses of a larger amplitude "B". (This single input-channel with dual-amplitude pulses was used to simplify the implementation of the experimental system.) The A-pulses include the quantized R_{pq} pulses ($R_{pq}+$ and $R_{pq}-$) and the R_{pk} compensation pulses which precede each quantized pulse by the interval τ_k; (see Chapter IV for the calculation of τ_k). A B-pulse replaces the coincident A-pulse whenever it is desired that the controlled variable reverse its previous direction of motion; also a B-pulse is inserted when it is desired that the controlled variable stop. The R_{pc} pulses of amplitude "A" or "B" are recorded as negative pulses on magnetic tape for storage; in the experimental system, a manual pulse-recording technique employing a push-button and a stop-watch was used.

The flow of signals within the reference-input circuit will now be described.

(1) During system operation, the R_{pc} pulses are directed into the inputs of two Schmitt triggers (ST-1 and ST-2) which emit positive pulses when triggered. ST-1 is triggered by pulses of either amplitude "A" or "B"; ST-2 is triggered only by pulses of the larger amplitude "B". Thus these Schmitt triggers "square-up" the tape recorder output pulses and sense the presence of a pulse of amplitude "B".

(2) The output pulses of ST-1 enter a 2-bit forward binary-counter consisting of two flip-flops (FF-2^0 and FF-2^1). As pulses are received, the counter, initially reset to the

0-state, assumes the states of 1, 2, 3, 0 in sequence and then repeats this sequence.

(3) The compensation integrators are controlled by the state of this counter as follows:

(a) state 0 -- integrator I_1 is reset and maintained at zero initial-conditions;

(b) state 0 and state 1 -- integrator I_3 is reset and maintained at zero initial-conditions; integrator I_4 integrates the output of integrator I_2, generating a R_Δ ramp-increment; (this output is zero if the system is at rest in state 0);

(c) state 1 -- integrator I_1 integrates its constant amplitude input -A during this interval τ_k, giving an output $A\tau_k$ which it retains during the following interval τ_q consisting of state 2 and state 3;

(d) state 2 -- integrator I_2 is reset and maintained at zero initial-conditions;

(e) state 2 and state 3 -- integrator I_4 is reset and maintained at zero initial-conditions; integrator I_3 integrates the output $A\tau_k$ of integrator I_1 generating a R_Δ ramp-increment;

(f) state 3 -- integrator I_2 integrates its constant amplitude input -A during this interval τ_k giving an output $A\tau_k$ which it retains during the following interval τ_q consisting of state 0 and state 1.

(4) The reference-pulse input R_p to the error bidirectional counter is generated by the one-zero transition of FF-2^0; thus alternate R_{pc} input pulses (the second, fourth, sixth, etc.) direct a pulse into the counter and change its state. Therefore the R_p pulse train is identical to the sum of the two R_{pq} pulse trains with two exceptions: (a) after the system has been at rest, the second R_{pc} pulse (first R_{pq} pulse) is prevented from generating a R_p pulse by FF-3 and its associated gates; (b) also an additonal R_p pulse is generated when a B-pulse stop-command is received, which commands the system to stop, and thus resets the circuit including FF-2^0 which experiences a one-zero transition.

(5) The pulse input R_p to the error bidirectional counter increases or decreases the number in the counter depending upon the state of FF-4. The error digital-analog converter then provides a step-varying analog output proportional to this

number. The reference-input component of this output is represented by $-R_d$ (the digital component of the continuous reference-input signal R_c).

(6) Summing amplifier S_1 adds the incremental component R_Δ and the digital component R_d and provides a resultant continuous reference-input signal R_c. This R_c is the component of the system error signal E due to the reference-input signal.

(7) The direction of change of the continuous reference-input signal R_c is controlled by FF-4 and its associated circuitry. When FF-4 is in state 0, a change occurring in R_c will be an increase; when it is in state 1, a change will be a decrease. When FF-4 is in state 0, the R_p pulses are directed into the plus input of the bidirectional counter and the R_Δ ramp increments are fed directly into the error summing amplifier S_1. When FF-4 is in state 1, the R_p pulses are directed into the minus input of the counter and the R_Δ ramp increments are first inverted by summing amplifier S_2 and then fed into the error summing amplifier S_1.

(8) A command for the system to stop or to reverse its direction of motion is indicated by the presence of a R_{pc} pulse

of amplitude B. A B-pulse triggers ST-1 and its output pulse enters the 2-bit forward counter and changes its state; this B-pulse also triggers ST-2 and its output pulse is momentarily delayed. If the counter is in state 1 or in state 3 when the delayed output pulse of ST-2 occurs, FF-3 and the counter flip-flops are reset to zero. The resulting one-zero transition of FF-2^0 directs a R_p pulse into the error bidirectional counter and the existence of these three flip-flops in the 0-state resets and maintains zero initial-conditions for the compensation integrators I_1, I_2, I_3, and I_4. If the counter is in state 0 or in state 2 when the delayed output pulse of ST-2 occurs, the state of FF-4 will be changed and a reversal in the direction of change of R_c will occur.

Therefore if the occurrence of a reference-input pulse R_{pc} of amplitude B shifts the counter into state 1 or state 3, it commands the system to stop; if it shifts the counter into state 0 or state 2, it commands the system to reverse its direction of motion. Note that when the system is initially reset manually, FF-4 is reset into state 0 and thus any succeeding change in R_c will be an increase until a reverse command is received.

(9) In the experimental system it was necessary to delay the input of the R_p pulse to the error bidirectional counter with a monostable multivibrator. This delay allowed the error digital-analog converter to change its output-state more nearly coincident with the reset operation of the integrators which was performed by electromechanical relays having an inherent delay in operation. A similar delay was also required by the quantizer feedback pulses θ_p.

It has been shown above that the reference-input circuit receives dual-amplitude input pulse information on a single input channel and converts it into a continuous reference input signal. This signal then commands the system to change the magnitude of the controlled variable by a specified amount and at a specified rate in a forward or reverse direction or commands it to stop.

System-Error Circuit

The continuous position-error signal E is obtained as the output of summing amplifier S_1. The error signal E is the result of the summation (and the inversion) of the following components: (a) the inverted digital error signal $-(R_d - \theta_d)$

which is the output of the system error digital-analog converter; this signal includes the inverted analog equivalent of the algebraic summation (by the error bidirectional counter and d-a converter) of the digital component R_d of the continuous reference-input signal R_c and of the digital component $-\theta_d$ of the continuous controlled-variable negative feedback signal $-\theta_c$; (b) the inverted reference compensation signal $-R_\Delta$; (c) the inverted negative-feedback compensation signal θ_Δ. Thus

$$
\begin{aligned}
E &= -\left\{\left[-(R_d - \theta_d)\right] + (-R_\Delta) + (\theta_\Delta)\right\} \\
&= (R_d + R_\Delta) - (\theta_d + \theta_\Delta) \\
&= R_c - \theta_c .
\end{aligned}
$$

Therefore E is a continuous error signal since it is the result of the algebraic summation of the continuous reference input signal R_c and the continuous controlled-variable negative feedback signal $-\theta_c$.

In order to make possible proportional-plus-integral error compensation, the position error E is integrated by integrator I_5 which gives an output $-K_I \int E\, dt$. This result is then inverted by summing amplifier S_3 giving the output $K_I \int E\, dt$ which is then combined with E in summer S_4.

Actuator Transfer Function

A typical actuator transfer function G(s) of the form

$$G(s) = \frac{K}{s(s^2 + 2\zeta s + 1)} ,$$

which includes one integration and an oscillatory term with damping ratio ζ of 0.6, was implemented with summing amplifiers S_4 and S_5 and integrators I_6, I_7, and I_8. Since $\theta(s)$ = E(s) G(s), these summers and integrators solved the following differential equation for θ and $\dot{\theta}$

$$\dddot{\theta} = KE - 2\zeta\ddot{\theta} - \dot{\theta} .$$

Thus this circuit provides output signals representing the controlled variable θ (position) and its time-rate of change $\dot{\theta}$ (velocity).

θ-Quantizer

Since the controlled variable θ was represented by a voltage, an electronic quantizer was employed to generate the quantized feedback pulses. The voltage θ is supplied as an input to summer S_6; the output of S_6 is fed directly into the input of Schmitt trigger ST-3 and into inverter S_7 which provides the input to ST-4. Now if θ increases by a one-quanta increment $\Delta\theta$ which gives a negative output from S_6 equal to the negative trigger-voltage of ST-3, then ST-3 emits a pulse. This pulse increases the number in the quantizer bidirectional counter and thus increases the magnitude of the negative output voltage of its associated digital-analog converter. This negative d-a output is fed back into the input of summer S_6 and amplified so as to provide a negative input which just cancels the positive θ-input. Thus the output of S_6 is returned to zero, ST-3 returns to its untriggered condition, and the process continues. The pulse emitted by ST-3 is also fed into the minus input of the system error bidirectional counter decreasing the number in the counter which represents the system digital position-error.

If, in a similar manner, θ decreases by a one-quanta increment $\Delta\theta$ which gives a negative output from S_7 equal to the negative trigger-voltage of ST-4, then ST-4 emits a pulse which decreases the magnitude of the negative output of the quantizer d-a converter and increases the number in the error bidirectional counter. The output of S_6 is returned to zero and ST-4 is returned to its untriggered condition.

Thus an increase (or decrease) in θ of an increment $\Delta\theta$ equal to one quanta will trigger ST-3 (or ST-4) and generate a quantized controlled-variable feedback pulse θ_p+ (or θ_p-) which enters the minus (or plus) input of the error bidirectional counter. The number in the counter, representing the digital position-error in quanta, is decreased (or increased) by one unit providing negative digital feedback.

Note that the feedback pulse-inhibiting circuit, which was shown in Chapter V to be required when an electromechanical quantizer is employed, was designed into the electronic quantizer used in the experimental system.

<u>θ_Δ-Compensator</u>

The compensation component θ_Δ of the continuous feedback signal θ_c is generated by integrators I_9 and I_{10}. The inverted controlled-variable velocity signal $-\dot{\theta}$ from the output of I_7 is integrated and inverted by integrators I_9 and I_{10} alternately during the interval between the generation of successive θ_p feedback pulses. The θ_p pulses alternately reset one integrator to zero initial-conditions while the other is integrating the input $-\dot{\theta}$ to produce a θ_Δ compensation increment. (Note that in an actual control system employing a mechanical actuator, the velocity signal $\dot{\theta}$ could be obtained from the output of a mechanically coupled dc tachometer.)

Continuous Controlled-Variable Negative Feedback Signal

The digital component θ_d of the continuous controlled-variable feedback signal θ_c is the component of the output of the system error digital-analog converter due to the input of the feedback pulses θ_p into the error bidirectional counter.

The incremental component θ_Δ of θ_c is the output of the θ_Δ-compensating integrators I_9 and I_{10}.

These components are added and inverted by summing amplifier S_1 giving the resulting continuous negative feedback signal $-\theta_c$ which is the component of the system error signal E due to controlled-variable negative feedback.

CHAPTER IX

OPERATION OF THE EXPERIMENTAL MODEL OF A COMPENSATED DIGITAL CONTROL SYSTEM

Desired Controlled-Variable Time-Response Curves

Four R_A-t curves, representing typical desired time-response curves (θ-t) for the controlled variable θ, are shown in Figure 9-1. These curves represent the following input commands to the controlled variable:

(a) Curve A -- a positive displacement of θ of 18 quanta at a constant velocity $\dot{\theta}$ of 0.1 quanta/second; a dwell of 10 seconds; a negative displacement of 15 quanta at 0.2 quanta/second; a dwell of 10 seconds; a positive displacement of 15 quanta at 0.1 quanta/second.

(b) Curve B -- a positive displacement of 15 quanta at 0.1 quanta/second; a dwell of 15 seconds; a negative displacement of 5 quanta at 0.2 quanta/second; a dwell of 10 seconds; a negative displacement of 5 quanta at 0.1 quanta/second; a positive

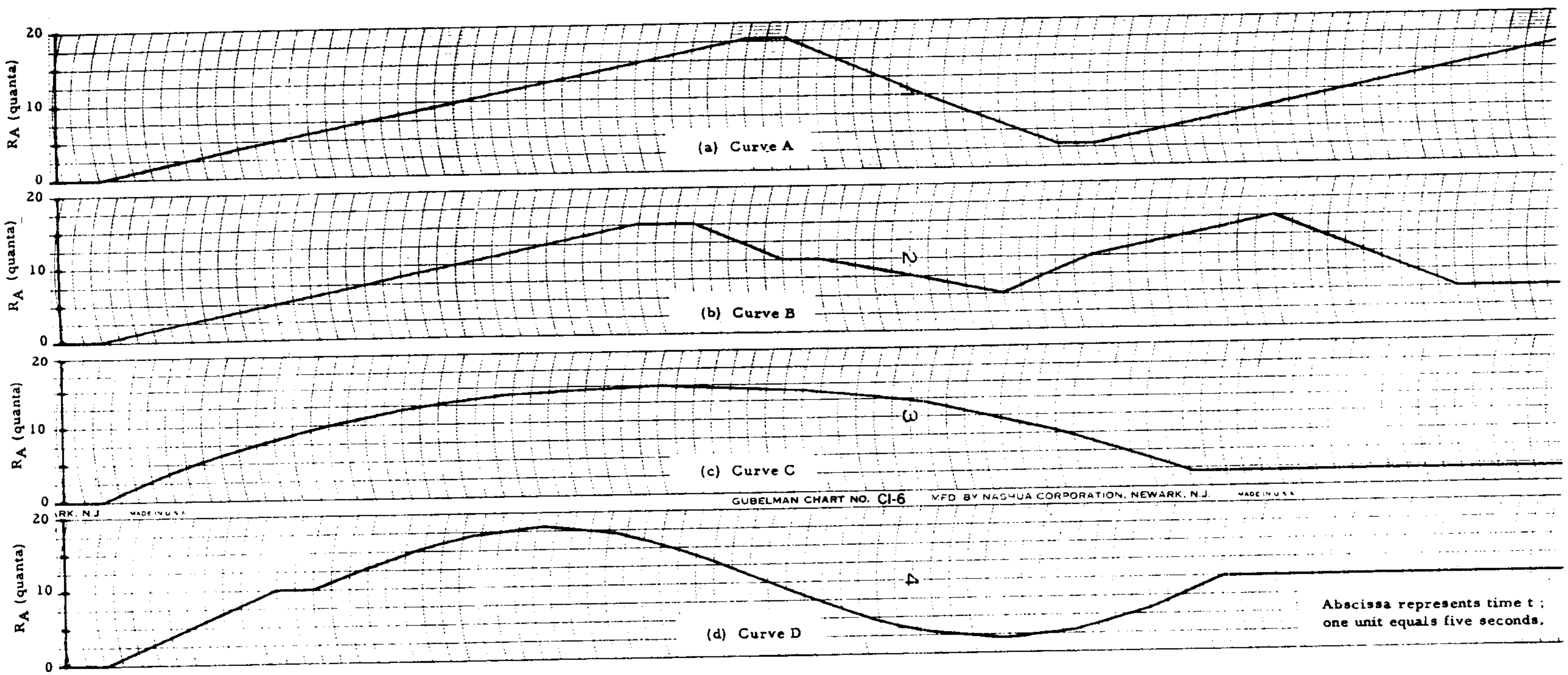


Figure 9-1. Typical <u>desired</u> controlled-variable time-response curves

displacement of 5 quanta at 0.2 quanta/second; a positive displacement of 5 quanta at 0.1 quanta/second; a negative displacement of 10 quanta at 0.2 quanta/second; a cessation of motion.

(c) Curve C -- a positive displacement to generate 179.55 degrees of arc of a circle of 209.79 quanta radius in 301.1 seconds; a cessation of motion; (the maximum displacement is 15 quanta and the final displacement is 2 quanta).

(d) Curve D -- a positive displacement of 10 quanta at 0.2 quanta/second; a dwell of 10 seconds; the generation of one cycle of a sine wave having an amplitude of 8 quanta and a period of 251.12 seconds; a cessation of motion; (the maximum displacement is 18 quanta and the final displacement is 10 quanta).

Compensated Reference-Input Pulses

These R_A-t curves were approximated by straight-line segments extending between quanta-point crossings. The resulting approximate-curves were then quantized, giving a pulse R_{pq} at the time of each quanta-point crossing. Compensation pulses R_{pk} were then added, giving compensated reference-input pulse information R_{pc} for each curve; see Figures 9-2, 9-3, 9-4, and 9-5.

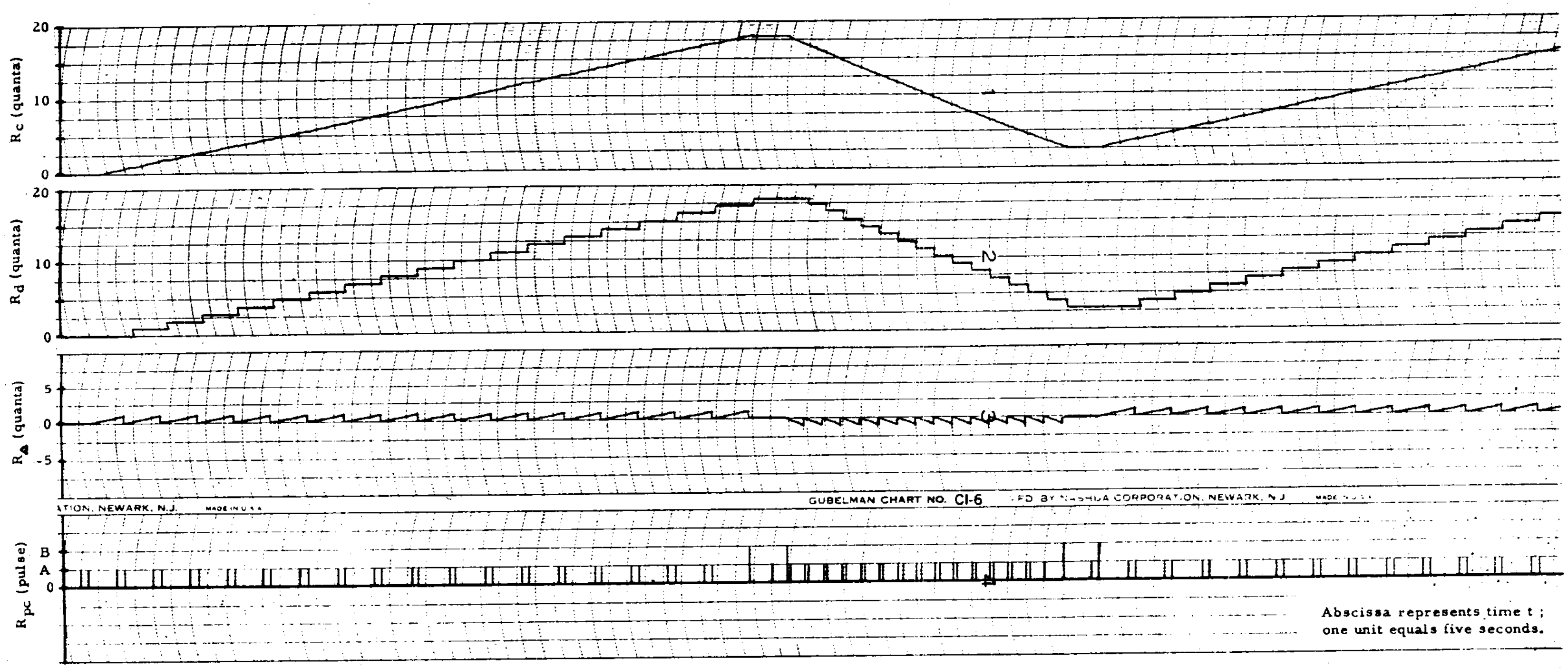


Figure 9-2. Continuous reference-input R_c and its components for the desired response of Curve A of Figure 9-1

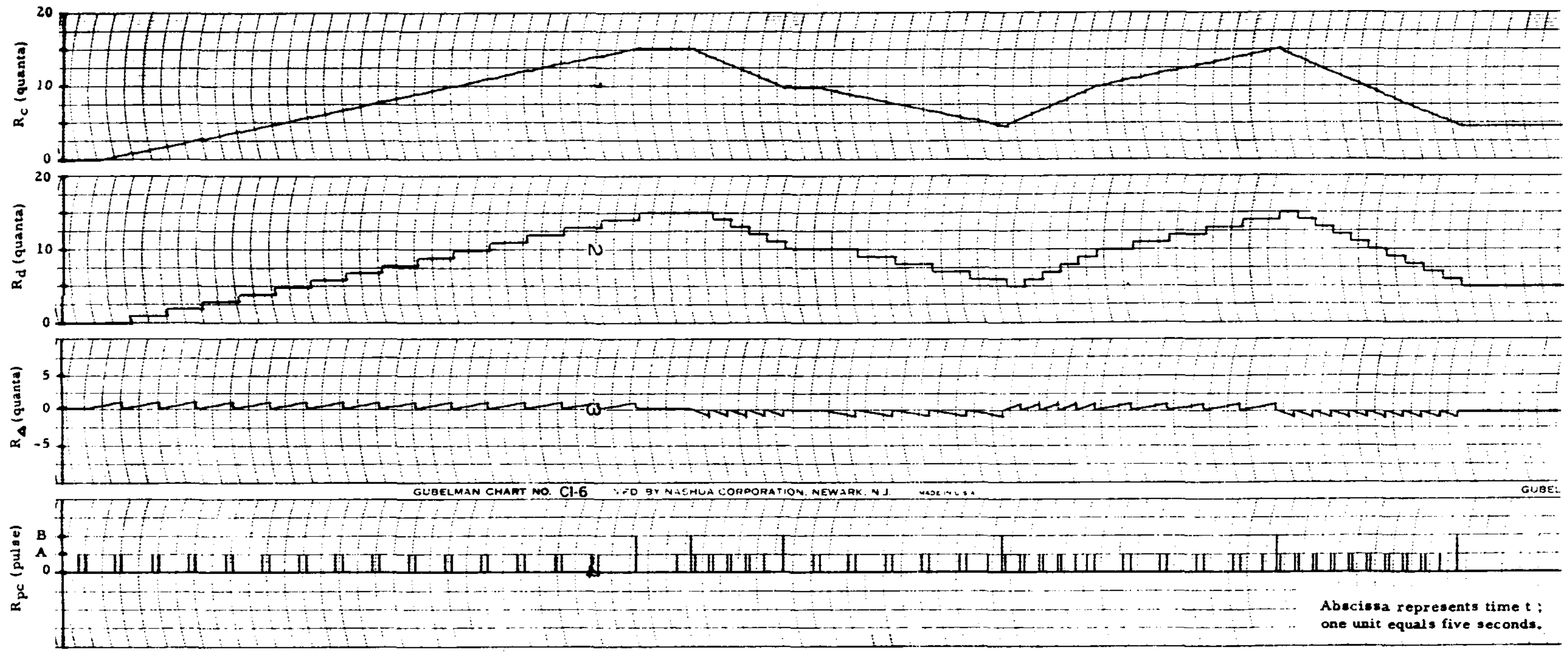


Figure 9-3. Continuous reference-input R_c and its components for the desired response of Curve B of Figure 9-1

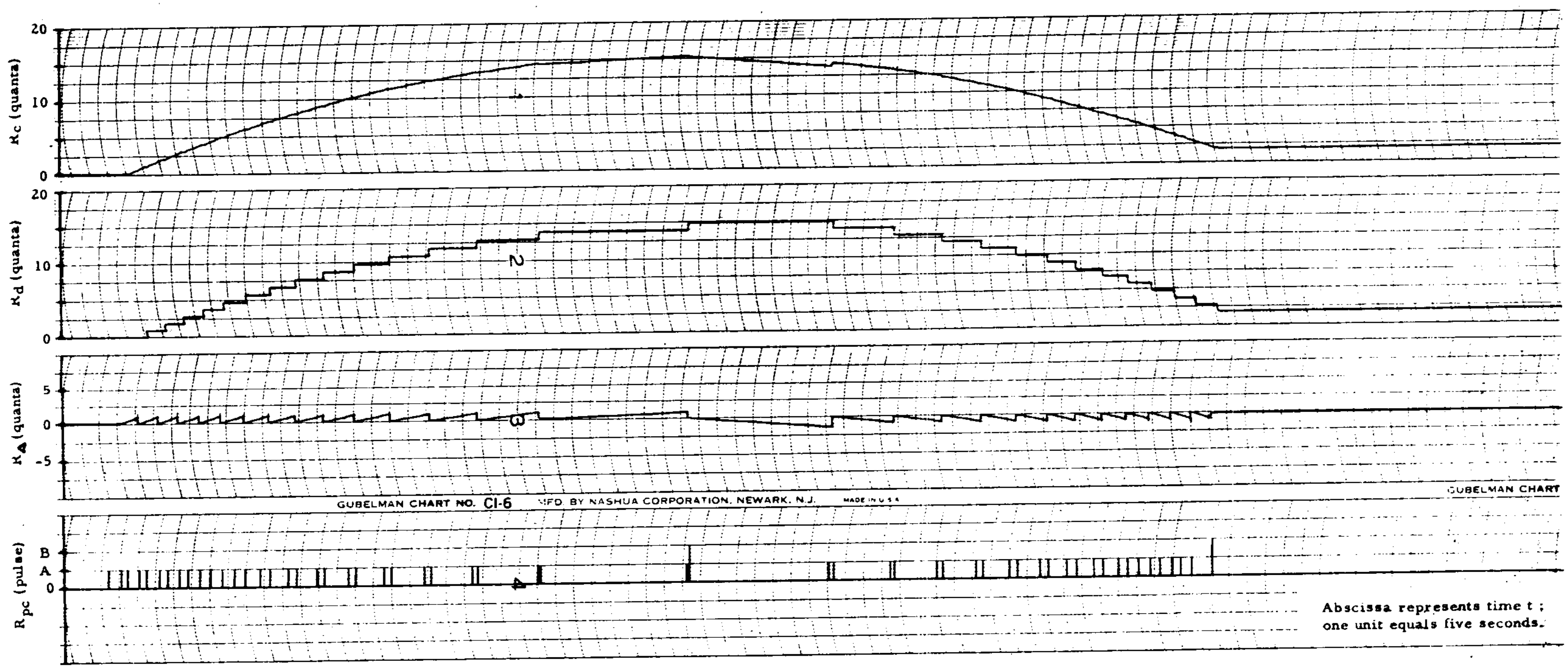


Figure 9-4. Continuous reference-input R_c and its components for the desired response of Curve C of Figure 9-1

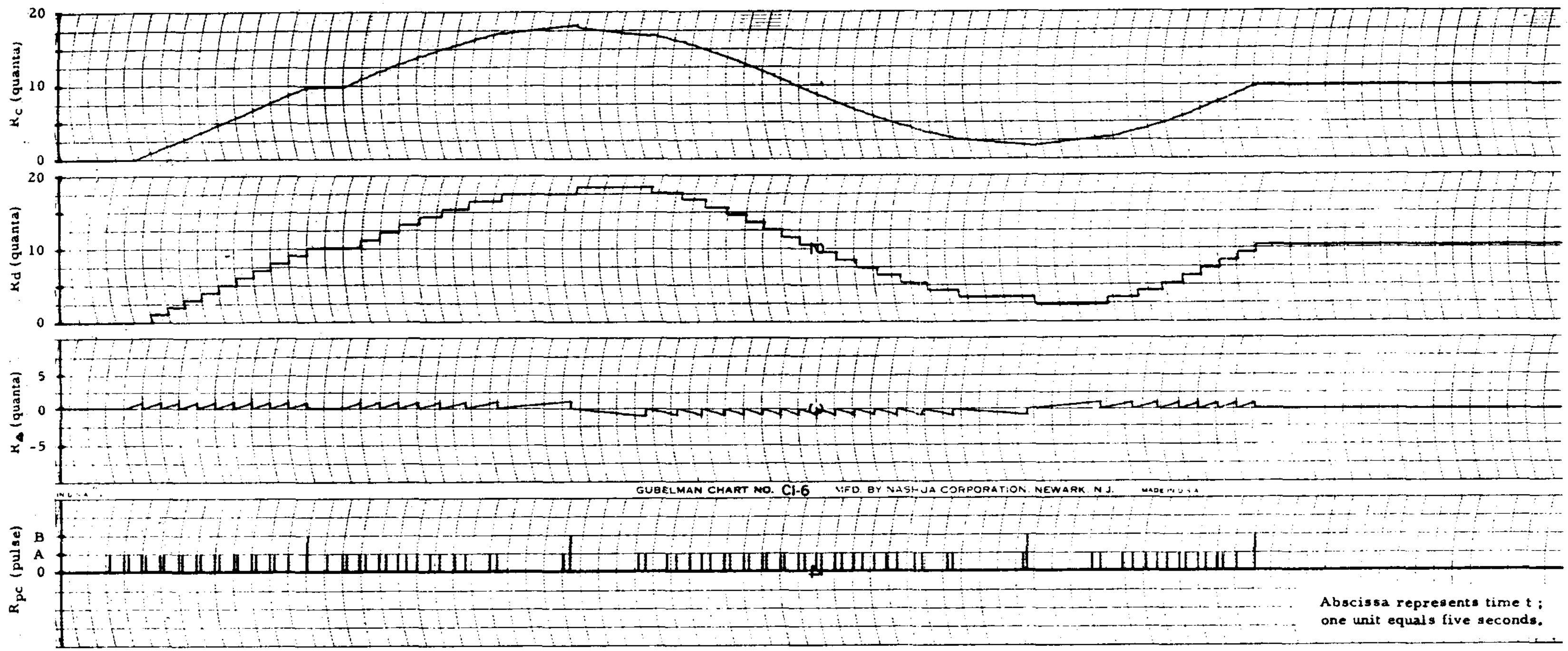


Figure 9-5. Continuous reference-input R_c and its components for the desired response of Curve D of Figure 9-1

Continuous Reference-Input Signal

The compensated reference-input pulses R_{pc} were recorded, in proper time relationship, on magnetic tape. The pulses were then fed into the input channel of the compensated control system. The pulses, the continuous reference-input signal R_c generated within the system, and its components were then recorded as a function of time. The continuous reference-input signal R_c, its digital component R_d, its incremental component R_Δ, and the associated compensated reference-input pulses R_{pc} used in its generation are shown in Figures 9-2, 9-3, 9-4, and 9-5 for the desired controlled-variable time-response of Curves A, B, C, and D, respectively, of Figure 9-1. Note that R_c is the algebraic sum of R_d and R_Δ as described in Chapter IV.

It can be seen, from Figure 9-1 through 9-5, that the compensated control system is capable of receiving input information R_{pc} in pulse form and of converting it into a continuous reference-input signal R_c which is identical in form to the straight-line approximation of the desired controlled-variable response R_A. Thus the compensated pulse-input system compensates itself into a continuous-input system; this system then has an input-signal accuracy which is equivalent to that of the uncompensated system at the time of the reception of the usual digital input pulses and which is better during the interval between these pulses.

Generation of the Continuous Controlled-Variable Feedback Signal

A typical controlled-variable time-response θ is shown in Figure 9-6. Also shown is the system's corresponding continuous feedback signal θ_c which is generated by the algebraic addition of its digital component θ_d and its incremental component θ_Δ; see Chapter V.

The digital component θ_d is the inverted analog output of the system's error bidirectional-counter and digital-analog converter due to the input of θ_p pulses only. The θ_p pulses are the θ_{pq} pulses, produced by quantizing θ, which are permitted to pass through the pulse-inhibiting circuit; see Chapter V. Note that, in the experimental system, the pulse-inhibiting circuit is designed into the θ-quantizer circuit.

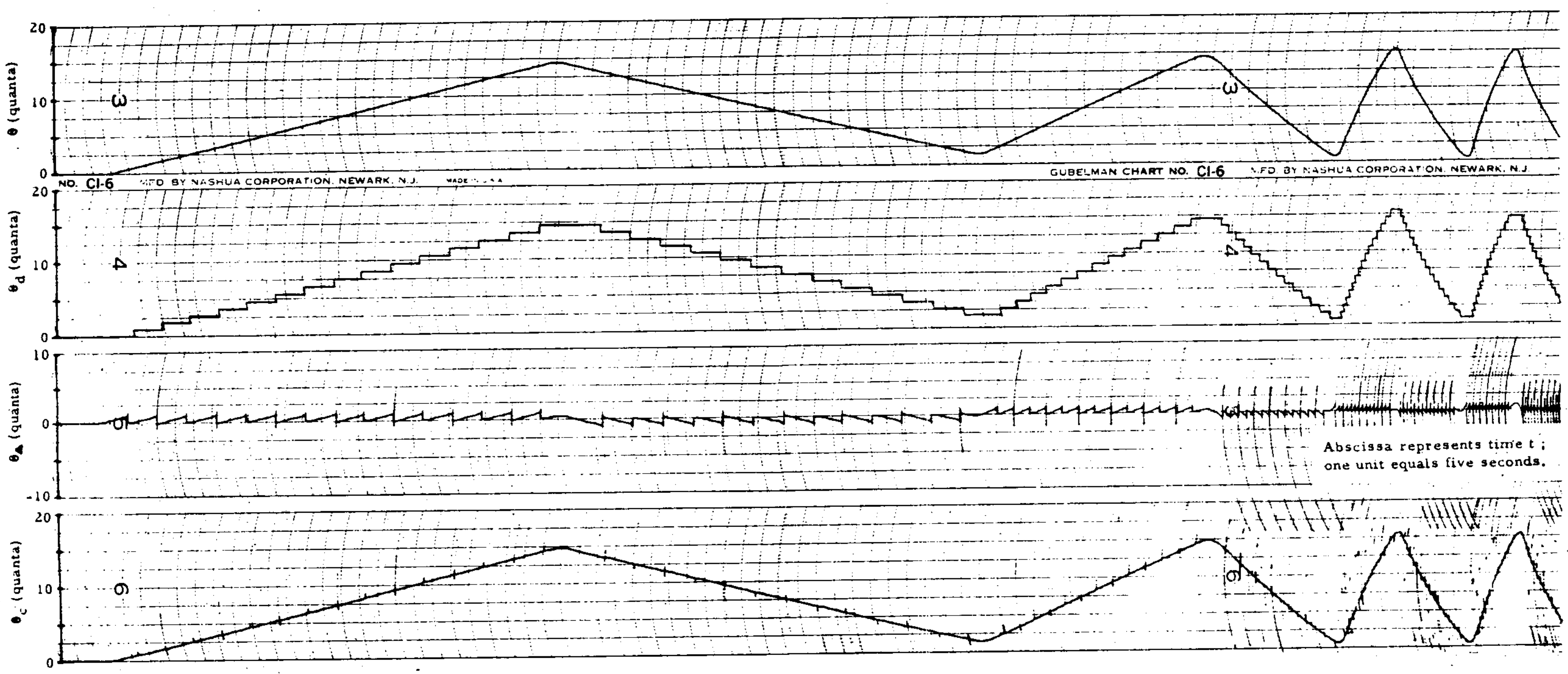


Figure 9-6. A typical controlled-variable time-response θ and the corresponding generated continuous feedback θ_c, including its components θ_d and $\theta_\blacktriangle$

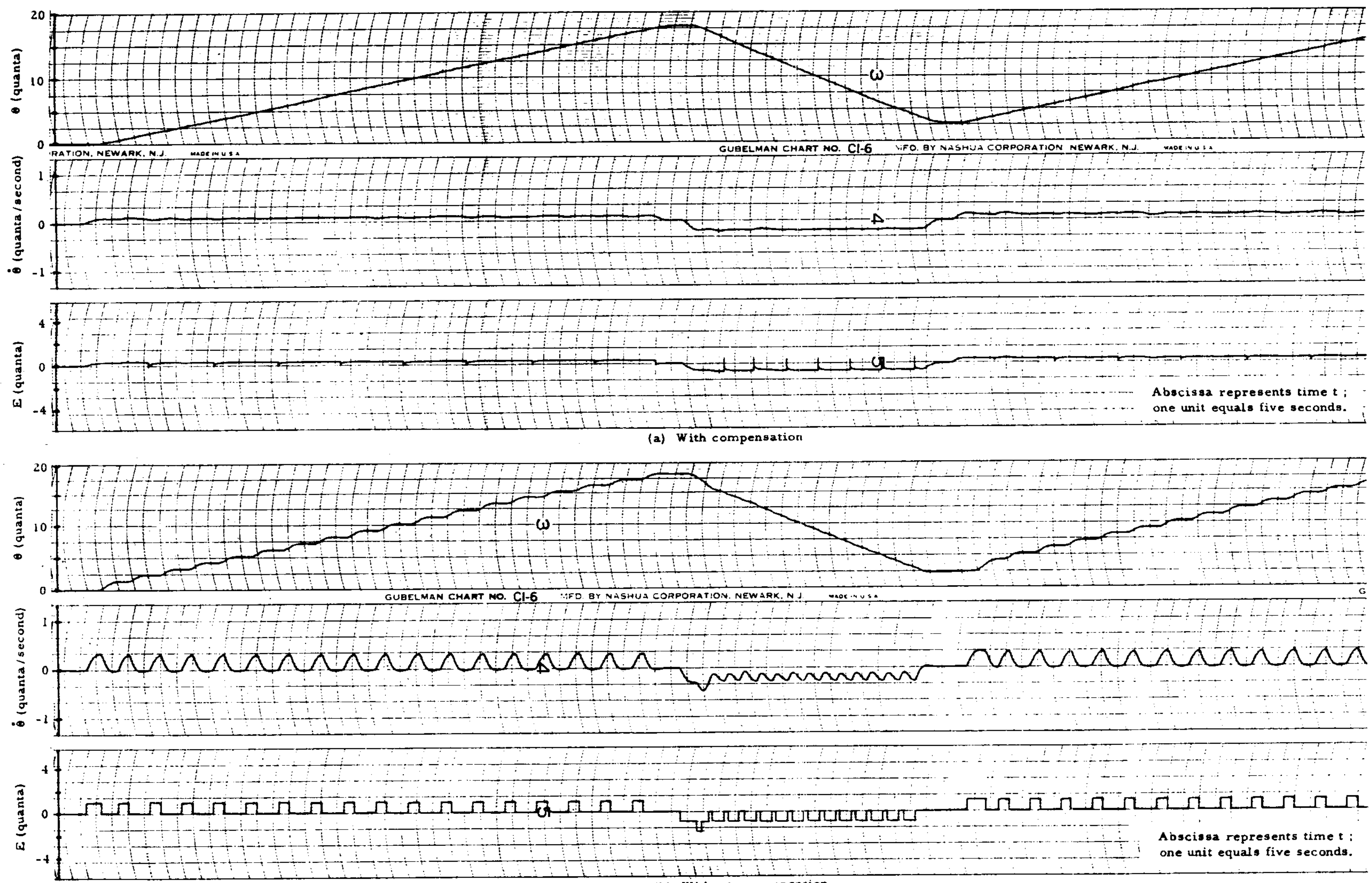
The incremental component θ_Δ is the output resulting from integrating the controlled-variable velocity $\dot{\theta}$ during the time interval between the generation of successive θ_p pulses (which reset the integrator). In a typical electromechanical system, $\dot{\theta}$ would be the output of a dc tachometer coupled to the controlled variable θ; however, in the experimental system, $\dot{\theta}$ was obtained as the output of an integrator included in the analog computer simulation of the transfer function G(s) of the system actuator.

By comparing θ_c with θ in Figure 9-6, it can be seen that a continuous feedback signal θ_c is generated by the algebraic addition of θ_d and θ_Δ; this signal has a form identical to that of θ. Thus the compensated system has a controlled-variable feedback signal θ_c available for negative feedback which is continuous, which has a form identical to that of θ, and which has the accuracy of the usual digital feedback signal at the time of occurrence of the digital feedback pulses θ_p and has a much better accuracy during the time interval between these pulses.

Time Response of the Controlled Variable for the Control System With and Without Compensation

The measured time response of the controlled variable θ for the experimental control system, with and without compensation, is shown in Figures 9-7, 9-8, 9-9, and 9-10 for the reference-input signal R_{pc} of Figures 9-2, 9-3, 9-4, and 9-5, respectively. Also shown are the controlled-variable velocity $\dot{\theta}$ and the system error signal E.

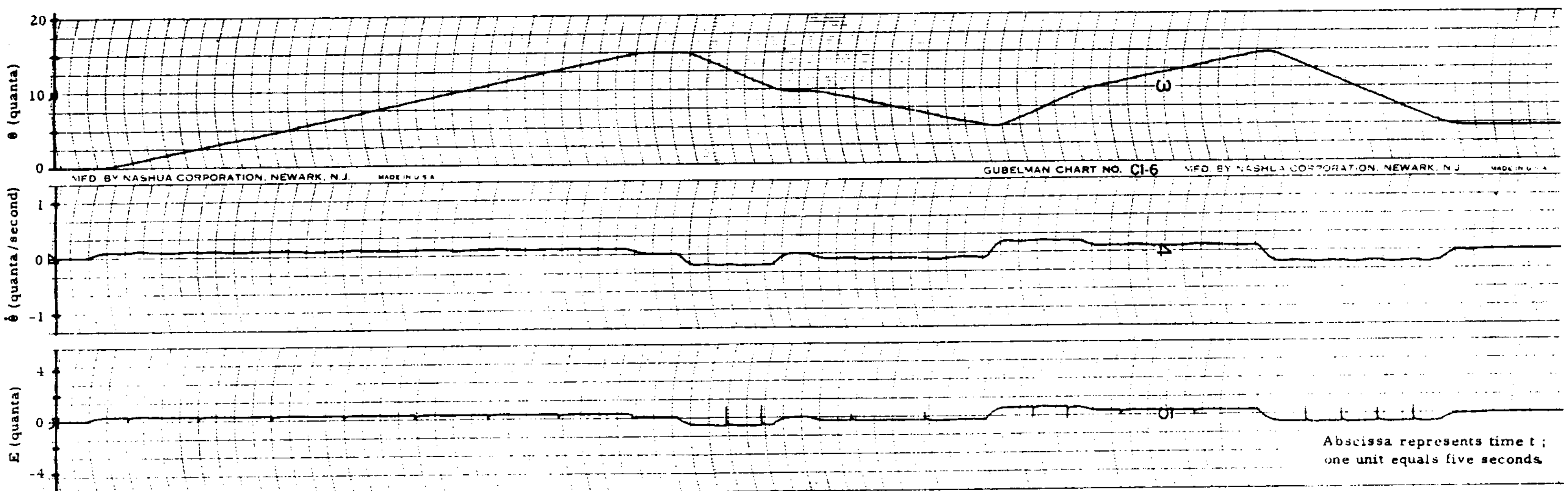
The compensated system has an actual controlled-variable response θ almost identical to the corresponding desired response R_A of Figure 9-1; also the velocity $\dot{\theta}$ and the error E are more nearly uniform. Thus it can be seen that these compensation techniques, which produce a continuous reference-input signal, a continuous controlled-variable feedback signal, and a resulting continuous system error signal, give a controlled-variable time-response that is superior to that of the system without compensation. (It should be noted that the discontinuities of very short duration which occur in the error signal E for the compensated system are due to the small time-delay



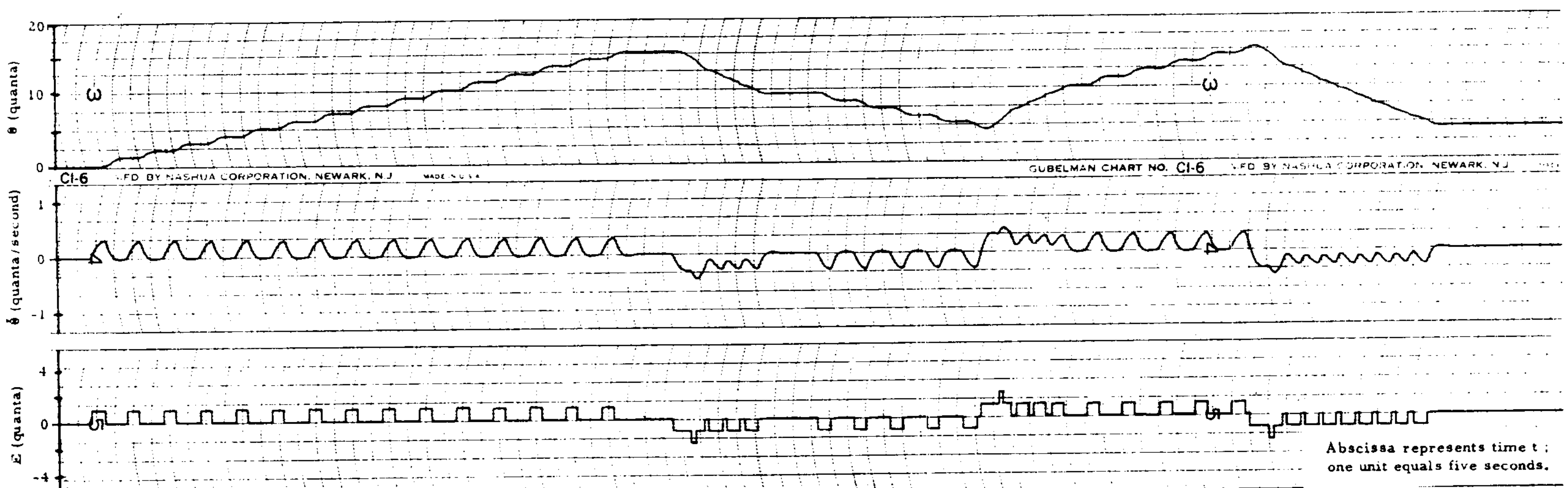
(a) With compensation

(b) Without compensation

Figure 9-7. Time response of the controlled variable θ for the experimental system with the reference-input R_{pc} of Figure 9-2

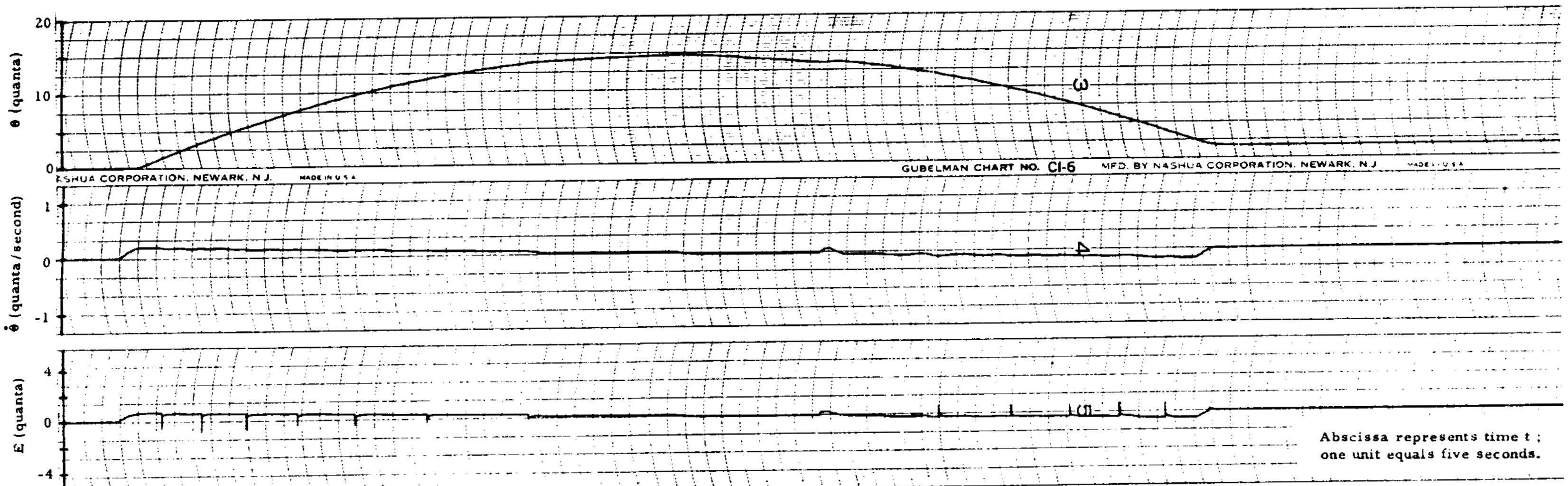


(a) With compensation

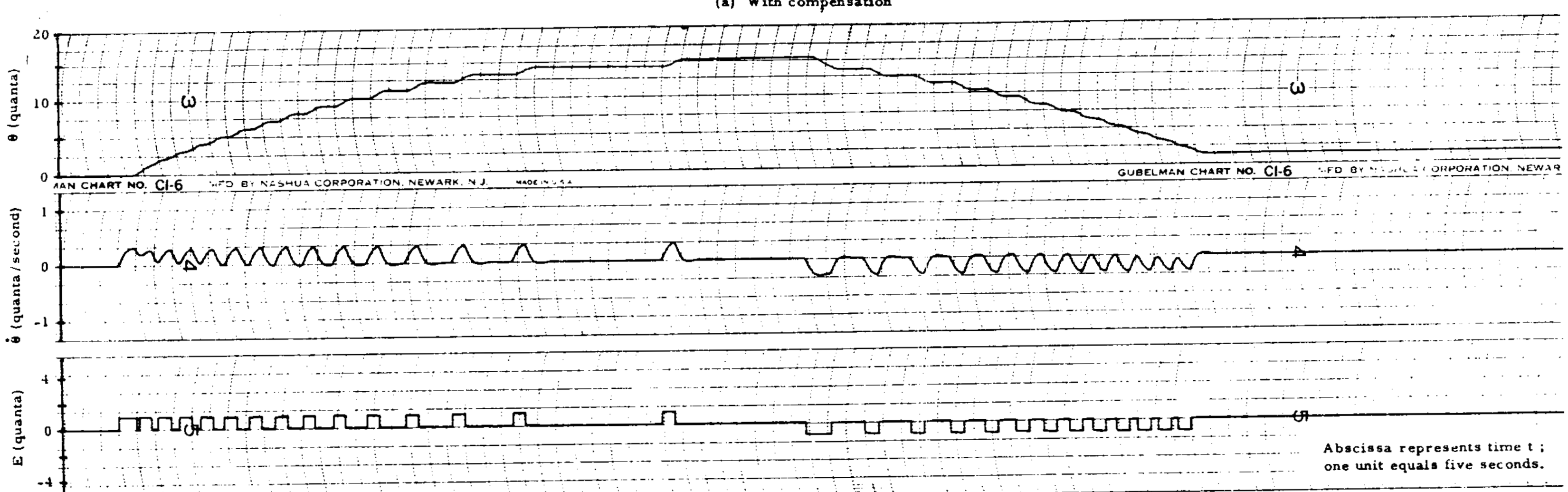


(b) Without compensation

Figure 9-8. Time response of the controlled variable θ for the experimental system with the reference-input R_{pc} of Figure 9-3



(a) With compensation



(b) Without compensation

Figure 9-9. Time response of the controlled variable θ for the experimental system with the reference-input R_{pc} of Figure 9-4

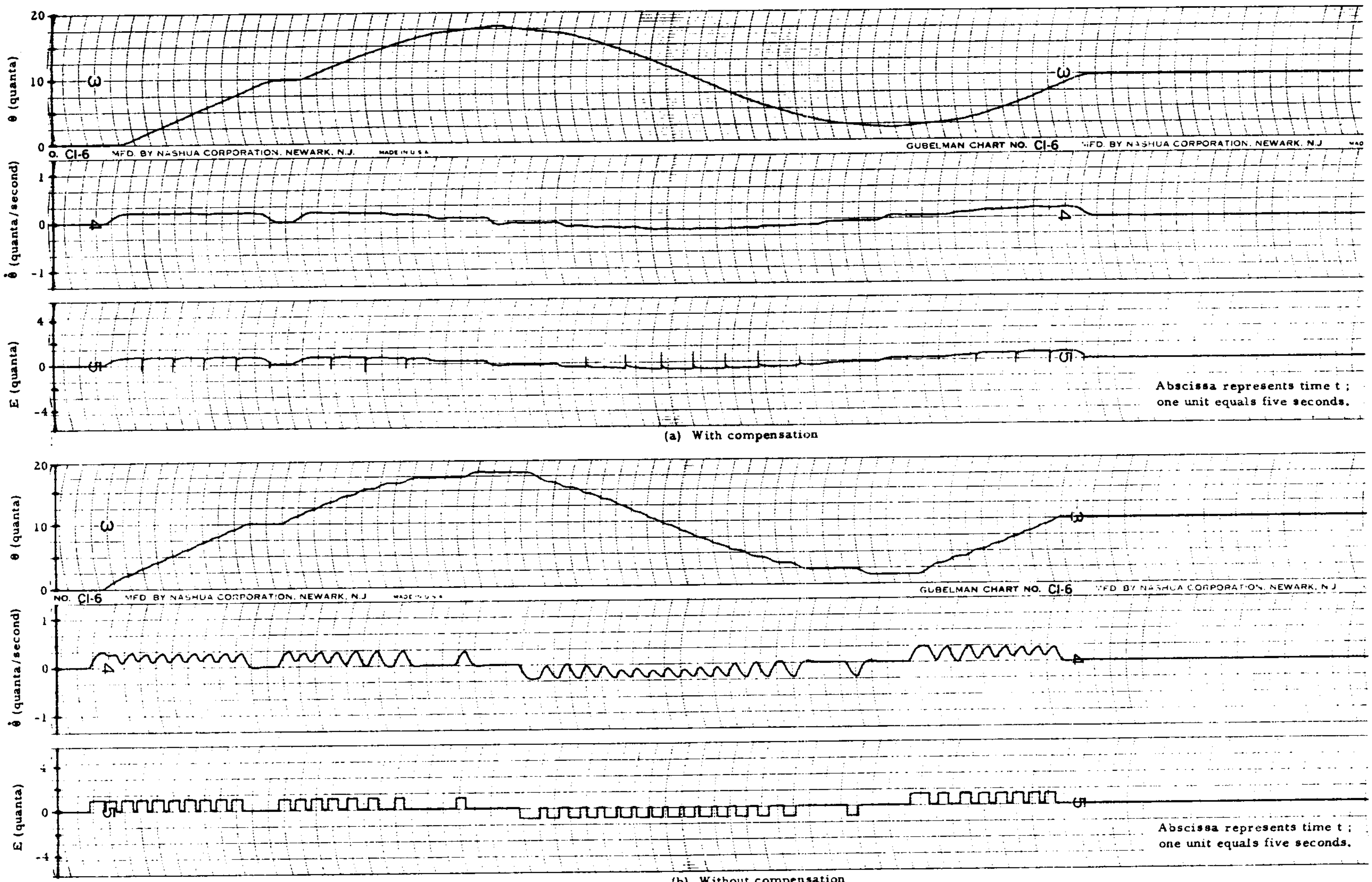


Figure 9-10. Time response of the controlled variable θ for the experimental system with the reference-input R_{pc} of Figure 9-5

in the operation of the electromechanical relays employed for resetting the system's compensation integrators. This delay could be practically eliminated by the substitution of faster-acting solid-state relays.)

Time Response of the Controlled Variable with Proportion-Plus-Integral Compensation

The time response of the controlled variable θ, its velocity $\dot{\theta}$, and the system error signal E are shown in Figure 9-11 for the compensated reference-input signal R_{pc} of Figure 9-2 and with proportional-plus-integral error compensation. An integration gain constant K_I of 0.05 was used. It can be seen that, for input signals commanding the controlled variable θ to move at a constant velocity (ramp inputs), the system error signal E decreases to zero soon after the initiation of the ramp. However, the transient condition in θ and $\dot{\theta}$ that follows the point of transition between ramps of different slopes is more pronounced in magnitude and extended in time duration as can be seen by the comparison of Figure 9-11 with Figure 9-7 for the compensated system without the addition of the proportional-plus-integral compensation.

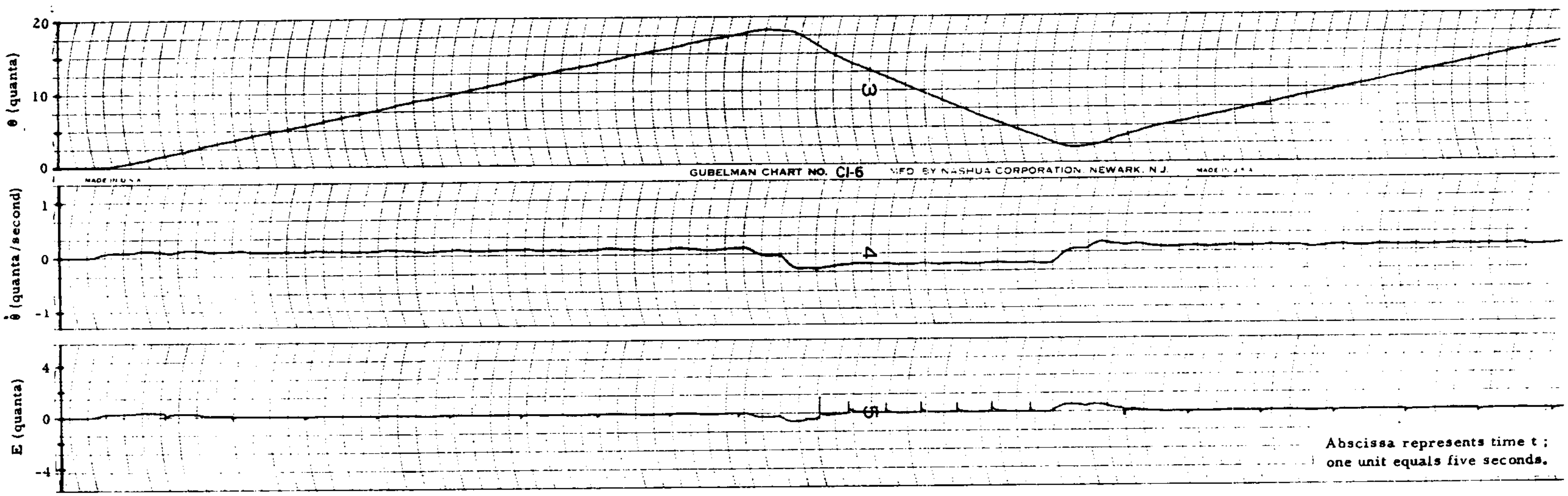


Figure 9-11. Time response of the controlled variable θ for the compensated experimental system with proportional-plus-integral error compensation and for the reference-input R_{pc} of Figure 9-2

CHAPTER X

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

Compensation of a pulse-input digital control system to generate a continuous reference-input signal, a continuous controlled-variable feedback signal, and a resulting continuous system error signal converts the discontinuous system into a system that is internally continuous and externally digital. This resulting continuous system has an accuracy that is equal to or better than that of the uncompensated digital system and a "smoothness" of operation comparable to that of a similar continuous control system. The actual response of the controlled variable θ follows the desired response R with better positional accuracy and with more nearly uniform velocity; these characteristics are very important for precision control of machine tools.

Recommendations

The design and installation of solid-state relays to replace the present electromechanical relays employed for resetting the compensation integrators would improve the experimental system's operation. Solid-state relays would practically eliminate the small discontinuities occurring in the system error signal due to the time delay in electromechanical relay operation; they would also make operation at higher input-pulse rates possible. The use of electromechanical relays was possible in the experimental system only because of the low input-pulse rates that were used. However, it is at low pulse-rates that a digital system demonstrates its most undesirable operational characteristics and thus requires compensation. In a practical system employing high pulse-frequencies, solid-state switching would be both desirable and essential.

The push-button and stop-watch technique that was used for recording the reference input pulses for the experimental system is applicable only to relatively slow input-pulse rates, and its accuracy is limited by human reaction time. Therefore, the design and construction of an electronic tape-input unit could facilitate the recording of these pulses on the magnetic-tape storage. Also a digital computer program could aid in the calculation of the time-location of the compensated reference-input pulses required to command the controlled variable to follow various desired time-response curves.

Some digital systems receive their reference-input information from punched paper tape on which have been coded the total desired displacement in quanta and the desired velocity in quanta/second for each individual ramp-response of the controlled variable. Therefore, in order to make the reference-input compensation circuitry compatible with digital control systems employing punched-paper-tape inputs, additional electronic circuitry could be utilized in the input channel of the system. These circuits would generate the compensated reference-input pulses R_{pc} from the coded displacement and rate data supplied by the punched paper tape.

Since analog signals, as well as the usual digital signals, are present in the compensated system, additional compensation techniques could be employed utilizing each of these signals independently to improve the system response during transient periods. A technique such as "digital-lead" compensation in conjunction with standard analog techniques might be used.

The design and utilization of solid-state operational amplifiers to perform the functions of integration, summation, and inversion required in the compensation circuitry would make possible superior system operation. The existence of undesirable pulses, extraneous signals, drift, and intermittent operational problems occuring with the electronic analog computer utilized in the construction of the experimental model made the system operation difficult and somewhat reduced its accuracy.

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APPENDIX

OPERATING CHARACTERISTICS OF CIRCUIT ELEMENTS

Bistable multivibrator. (a) A positive voltage-change at input R triggers (or maintains) the circuit into state-0; thereafter a negative voltage exists at output 0 and a zero voltage exists at output 1. (b) A positive voltage-change at input S triggers (or maintains) the circuit into state-1 and a zero voltage exists at output 0. (c) A positive voltage-change at input T triggers the circuit from its present state into the alternate state.

Nor gate. The output of O is a negative voltage whenever all inputs (A, B, C, ...) are at zero voltage; otherwise the output of O is zero voltage.

Gated pulse-generator. The output of P_o will be a negative-going pulse if a zero voltage exists at "level" input L whenever a positive voltage-change occurs at "pulse" input P.

Monostable multivibrator. In the steady-state, a negative voltage exists at output 0 and a zero voltage exists at output 1. Whenever a positive voltage-change occurs at input I, the circuit is triggered into the alternate state (zero voltage at output 0 and negative voltage at output 1); after a delay-time of T the circuit returns to its initial condition.

Astable multivibrator. Output 0 and output 1 alternate between zero voltage and a negative voltage in a step-like manner at frequency f.

Or gate. An output exists at output O whenever an input exists at any input (A, B, C, ...); otherwise the output is zero.

And gate. An output exists at output O whenever an input exists at all inputs (A, B, C, ...); otherwise the output is zero.

Schmitt trigger. If input I is initially a zero voltage, output T is a zero voltage and output S is a negative voltage; if input I becomes more negative than -U volts, then S and T switch states and retain these new states until input I becomes less negative than -L volts; then S and T return to their initial states.

<u>Gate (transistor).</u> When control-input C is a zero voltage, A and B are open-circuited; when C is a negative saturation voltage, A and B are connected so that current can flow from A <u>to</u> B <u>only</u>.

<u>Relay (electromechanical).</u> When control input C is a zero voltage, A and B are open-circuited; when C is a negative magnetizing voltage, A and B are connected so that current can flow between A and B in <u>either</u> direction.

<u>Summing amplifier.</u> Output O is equal to the algebraic sum of inputs A, B, and C ($O = A + B - C$).

<u>Summing amplifier (and inverter).</u> Output O is equal to the inverted algebraic sum of inputs A, B, and C ($O = -A - B + C$).

<u>Integrator (and inverter).</u> Output O is equal to the inverted integral of input I ($O = -\int I\,dt$); input R resets the integrator to zero initial conditions.